

# 1 Linear Functions

- 1.1 Parent Functions and Transformations
- 1.2 Transformations of Linear and Absolute Value Functions
- 1.3 Modeling with Linear Functions
- 1.4 Solving Linear Systems



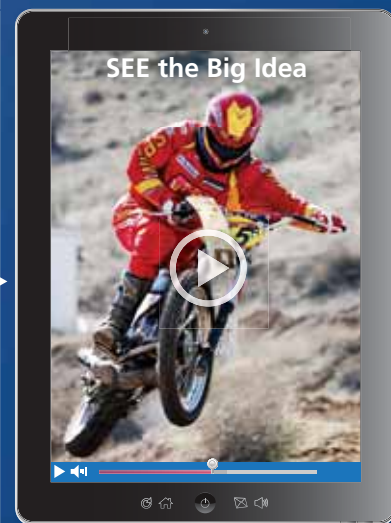
Pizza Shop (p. 34)



Prom (p. 23)



Café Expenses (p. 16)



Dirt Bike (p. 7)



Swimming (p. 10)

# Maintaining Mathematical Proficiency

## Evaluating Expressions

**Example 1** Evaluate the expression  $36 \div (3^2 \times 2) - 3$ .

$$\begin{aligned} 36 \div (3^2 \times 2) - 3 &= 36 \div (9 \times 2) - 3 \\ &= 36 \div 18 - 3 \\ &= 2 - 3 \\ &= -1 \end{aligned}$$

Evaluate the power within parentheses.

Multiply within parentheses.

Divide.

Subtract.

Evaluate.

1.  $5 \cdot 2^3 + 7$

2.  $4 - 2(3 + 2)^2$

3.  $48 \div 4^2 + \frac{3}{5}$

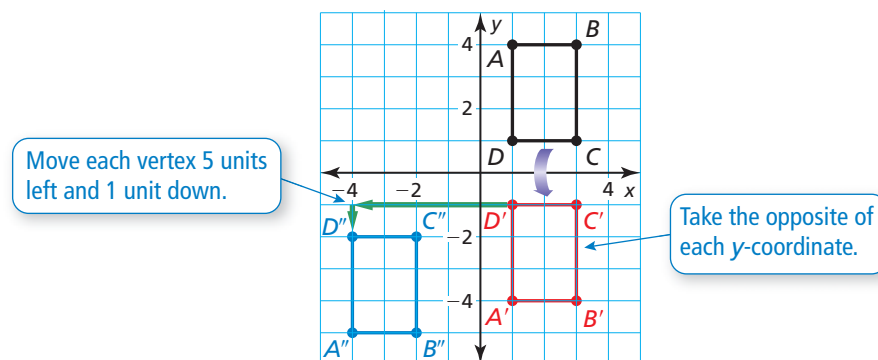
4.  $50 \div 5^2 \cdot 2$

5.  $\frac{1}{2}(2^2 + 22)$

6.  $\frac{1}{6}(6 + 18) - 2^2$

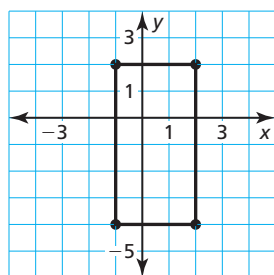
## Transformations of Figures

**Example 2** Reflect the black rectangle in the  $x$ -axis. Then translate the new rectangle 5 units to the left and 1 unit down.

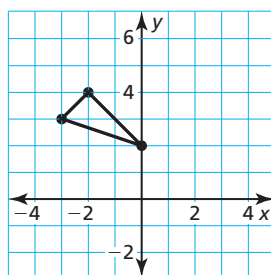


Graph the transformation of the figure.

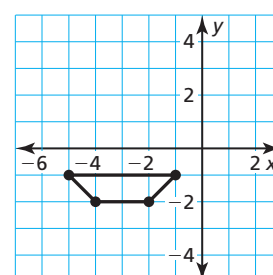
7. Translate the rectangle 1 unit right and 4 units up.



8. Reflect the triangle in the  $y$ -axis. Then translate 2 units left.



9. Translate the trapezoid 3 units down. Then reflect in the  $x$ -axis.



10. **ABSTRACT REASONING** Give an example to show why the order of operations is important when evaluating a numerical expression. Is the order of transformations of figures important? Justify your answer.

# Mathematical Practices

Mathematically proficient students use technological tools to explore concepts.

## Using a Graphing Calculator

### Core Concept

#### Standard and Square Viewing Windows

A typical screen on a graphing calculator has a height-to-width ratio of 2 to 3. This means that when you view a graph using the *standard viewing window* of  $-10$  to  $10$  (on each axis), the graph will not be shown in its true perspective.

To view a graph in its true perspective, you need to change to a *square viewing window*, where the tick marks on the  $x$ -axis are spaced the same as the tick marks on the  $y$ -axis.

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
    
```

This is the standard viewing window.

```

WINDOW
Xmin=-9
Xmax=9
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
    
```

This is a square viewing window.

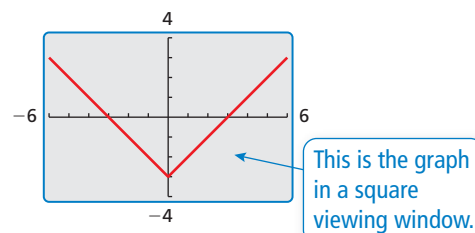
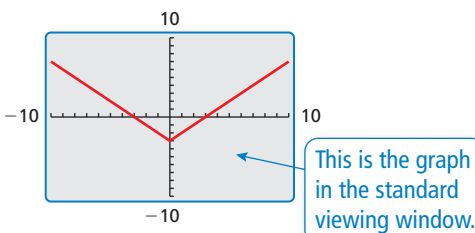
#### EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to graph  $y = |x| - 3$ .

#### SOLUTION

In the standard viewing window, notice that the tick marks on the  $y$ -axis are closer together than those on the  $x$ -axis. This implies that the graph is not shown in its true perspective.

In a square viewing window, notice that the tick marks on both axes have the same spacing. This implies that the graph is shown in its true perspective.



## Monitoring Progress

Use a graphing calculator to graph the equation using the standard viewing window and a square viewing window. Describe any differences in the graphs.

- $y = 2x - 3$
- $y = |x + 2|$
- $y = -x^2 + 1$
- $y = \sqrt{x - 1}$
- $y = x^3 - 2$
- $y = 0.25x^3$

Determine whether the viewing window is square. Explain.

- $-8 \leq x \leq 8, -2 \leq y \leq 8$
- $-7 \leq x \leq 8, -2 \leq y \leq 8$
- $-6 \leq x \leq 9, -2 \leq y \leq 8$
- $-2 \leq x \leq 2, -3 \leq y \leq 3$
- $-4 \leq x \leq 5, -3 \leq y \leq 3$
- $-4 \leq x \leq 4, -3 \leq y \leq 3$

# 1.1 Parent Functions and Transformations

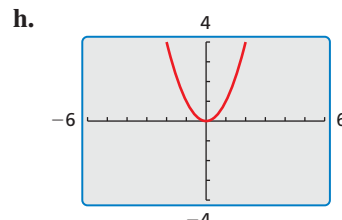
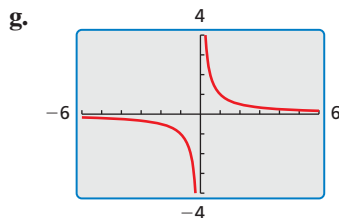
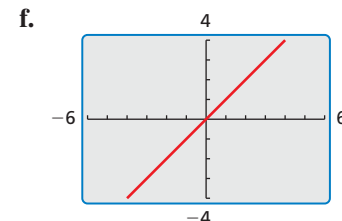
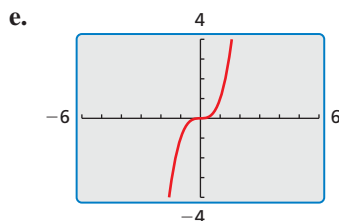
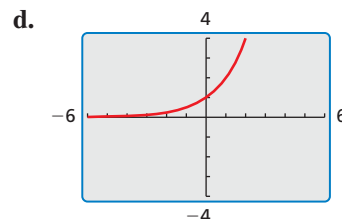
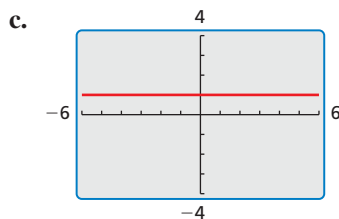
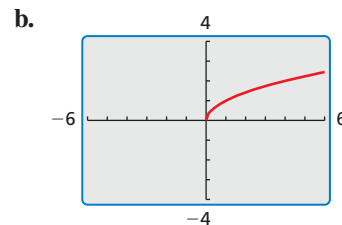
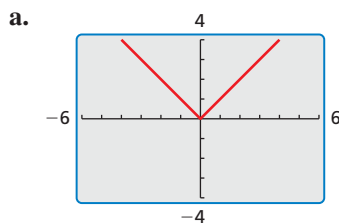
**Essential Question** What are the characteristics of some of the basic parent functions?

## EXPLORATION 1 Identifying Basic Parent Functions

**Work with a partner.** Graphs of eight basic parent functions are shown below. Classify each function as *constant*, *linear*, *absolute value*, *quadratic*, *square root*, *cubic*, *reciprocal*, or *exponential*. Justify your reasoning.

### JUSTIFYING CONCLUSIONS

To be proficient in math, you need to justify your conclusions and communicate them clearly to others.



## Communicate Your Answer

2. What are the characteristics of some of the basic parent functions?
3. Write an equation for each function whose graph is shown in Exploration 1. Then use a graphing calculator to verify that your equations are correct.

# 1.1 Lesson

## Core Vocabulary

parent function, p. 4  
 transformation, p. 5  
 translation, p. 5  
 reflection, p. 5  
 vertical stretch, p. 6  
 vertical shrink, p. 6

### Previous

function  
 domain  
 range  
 slope  
 scatter plot

## What You Will Learn

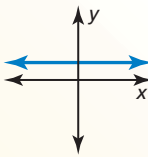
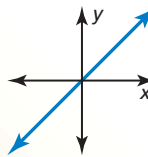
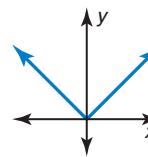
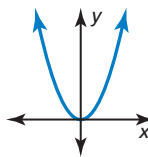
- ▶ Identify families of functions.
- ▶ Describe transformations of parent functions.
- ▶ Describe combinations of transformations.

## Identifying Function Families

Functions that belong to the same *family* share key characteristics. The **parent function** is the most basic function in a family. Functions in the same family are *transformations* of their parent function.

## Core Concept

### Parent Functions

Family	Constant	Linear	Absolute Value	Quadratic
<b>Rule</b>	$f(x) = 1$	$f(x) = x$	$f(x) =  x $	$f(x) = x^2$
<b>Graph</b>				
<b>Domain</b>	All real numbers	All real numbers	All real numbers	All real numbers
<b>Range</b>	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

## LOOKING FOR STRUCTURE

You can also use function rules to identify functions. The only variable term in  $f$  is an  $|x|$ -term, so it is an absolute value function.

### EXAMPLE 1

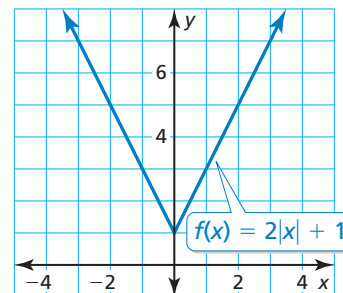
### Identifying a Function Family

Identify the function family to which  $f$  belongs. Compare the graph of  $f$  to the graph of its parent function.

### SOLUTION

The graph of  $f$  is V-shaped, so  $f$  is an absolute value function.

The graph is shifted up and is narrower than the graph of the parent absolute value function. The domain of each function is all real numbers, but the range of  $f$  is  $y \geq 1$  and the range of the parent absolute value function is  $y \geq 0$ .

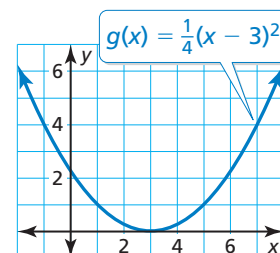


## Monitoring Progress



Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

1. Identify the function family to which  $g$  belongs. Compare the graph of  $g$  to the graph of its parent function.



## Describing Transformations

A **transformation** changes the size, shape, position, or orientation of a graph.

A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

### REMEMBER

The slope-intercept form of a linear equation is  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept.

### EXAMPLE 2

**Graphing and Describing Translations**

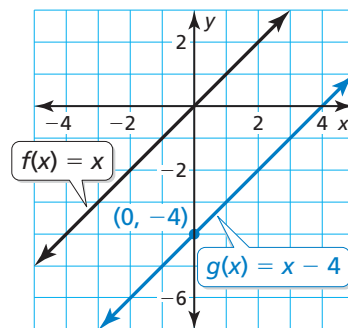
Graph  $g(x) = x - 4$  and its parent function. Then describe the transformation.

#### SOLUTION

The function  $g$  is a linear function with a slope of 1 and a  $y$ -intercept of  $-4$ . So, draw a line through the point  $(0, -4)$  with a slope of 1.

The graph of  $g$  is 4 units below the graph of the parent linear function  $f$ .

► So, the graph of  $g(x) = x - 4$  is a vertical translation 4 units down of the graph of the parent linear function.



A **reflection** is a transformation that flips a graph over a line called the *line of reflection*. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

### EXAMPLE 3

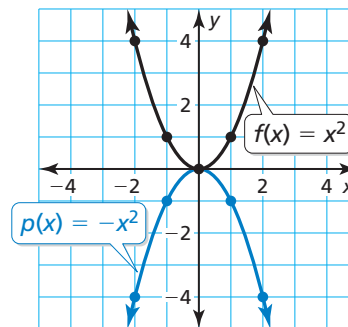
**Graphing and Describing Reflections**

Graph  $p(x) = -x^2$  and its parent function. Then describe the transformation.

#### SOLUTION

The function  $p$  is a quadratic function. Use a table of values to graph each function.

$x$	$y = x^2$	$y = -x^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4



The graph of  $p$  is the graph of the parent function flipped over the  $x$ -axis.

► So,  $p(x) = -x^2$  is a reflection in the  $x$ -axis of the parent quadratic function.

### REMEMBER

The function  $p(x) = -x^2$  is written in *function notation*, where  $p(x)$  is another name for  $y$ .

## Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Graph the function and its parent function. Then describe the transformation.

2.  $g(x) = x + 3$

3.  $h(x) = (x - 2)^2$

4.  $n(x) = -|x|$



Another way to transform the graph of a function is to multiply all of the  $y$ -coordinates by the same positive factor (other than 1). When the factor is greater than 1, the transformation is a **vertical stretch**. When the factor is greater than 0 and less than 1, it is a **vertical shrink**.

### EXAMPLE 4 Graphing and Describing Stretches and Shrinks

Graph each function and its parent function. Then describe the transformation.

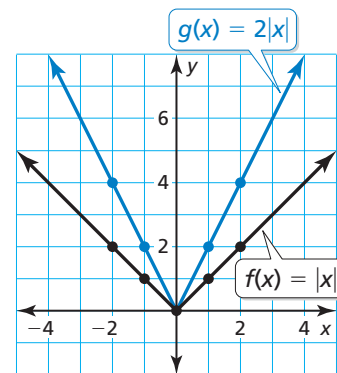
a.  $g(x) = 2|x|$

b.  $h(x) = \frac{1}{2}x^2$

#### SOLUTION

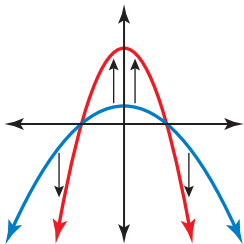
- a. The function  $g$  is an absolute value function. Use a table of values to graph the functions.

$x$	$y =  x $	$y = 2 x $
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4

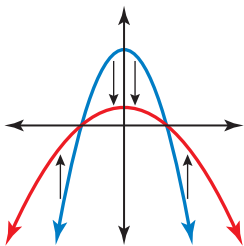


#### REASONING ABSTRACTLY

To visualize a vertical stretch, imagine *pulling* the points away from the  $x$ -axis.



To visualize a vertical shrink, imagine *pushing* the points toward the  $x$ -axis.

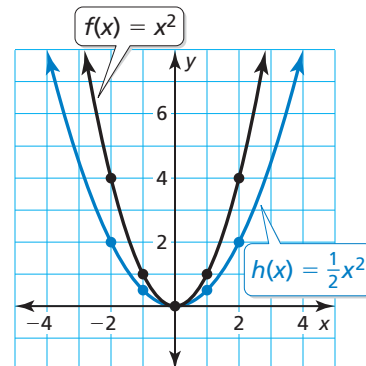


The  $y$ -coordinate of each point on  $g$  is two times the  $y$ -coordinate of the corresponding point on the parent function.

- So, the graph of  $g(x) = 2|x|$  is a vertical stretch of the graph of the parent absolute value function.

- b. The function  $h$  is a quadratic function. Use a table of values to graph the functions.

$x$	$y = x^2$	$y = \frac{1}{2}x^2$
-2	4	2
-1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	4	2



The  $y$ -coordinate of each point on  $h$  is one-half of the  $y$ -coordinate of the corresponding point on the parent function.

- So, the graph of  $h(x) = \frac{1}{2}x^2$  is a vertical shrink of the graph of the parent quadratic function.

#### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Graph the function and its parent function. Then describe the transformation.

5.  $g(x) = 3x$

6.  $h(x) = \frac{3}{2}x^2$

7.  $c(x) = 0.2|x|$

## Combinations of Transformations

You can use more than one transformation to change the graph of a function.

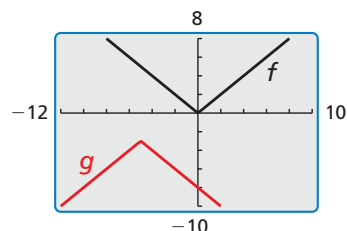
### EXAMPLE 5 Describing Combinations of Transformations

Use a graphing calculator to graph  $g(x) = -|x + 5| - 3$  and its parent function. Then describe the transformations.

#### SOLUTION

The function  $g$  is an absolute value function.

- The graph shows that  $g(x) = -|x + 5| - 3$  is a reflection in the  $x$ -axis followed by a translation 5 units left and 3 units down of the graph of the parent absolute value function.



### EXAMPLE 6 Modeling with Mathematics

Time (seconds), $x$	Height (feet), $y$
0	8
0.5	20
1	24
1.5	20
2	8

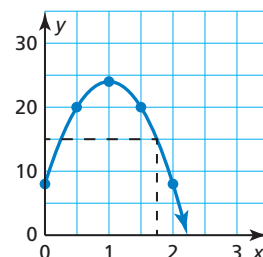
The table shows the height  $y$  of a dirt bike  $x$  seconds after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds.

#### SOLUTION

- Understand the Problem** You are asked to identify the type of function that can model the table of values and then to find the height at a specific time.
- Make a Plan** Create a scatter plot of the data. Then use the relationship shown in the scatter plot to estimate the height after 1.75 seconds.
- Solve the Problem** Create a scatter plot.

The data appear to lie on a curve that resembles a quadratic function. Sketch the curve.

- So, you can model the data with a quadratic function. The graph shows that the height is about 15 feet after 1.75 seconds.



- Look Back** To check that your solution is reasonable, analyze the values in the table. Notice that the heights decrease after 1 second. Because 1.75 is between 1.5 and 2, the height must be between 20 feet and 8 feet.

$$8 < 15 < 20 \quad \checkmark$$



### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Use a graphing calculator to graph the function and its parent function. Then describe the transformations.

8.  $h(x) = -\frac{1}{4}x + 5$

9.  $d(x) = 3(x - 5)^2 - 1$

10. The table shows the amount of fuel in a chainsaw over time. What type of function can you use to model the data? When will the tank be empty?

Time (minutes), $x$	0	10	20	30	40
Fuel remaining (fluid ounces), $y$	15	12	9	6	3



## Vocabulary and Core Concept Check

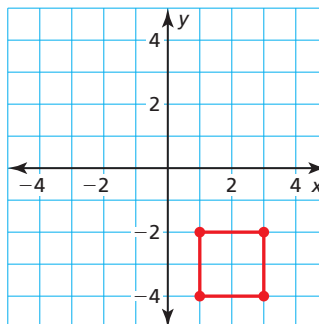
- COMPLETE THE SENTENCE** The function  $f(x) = x^2$  is the \_\_\_\_\_ of  $f(x) = 2x^2 - 3$ .
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What are the vertices of the figure after a reflection in the  $x$ -axis, followed by a translation 2 units right?

What are the vertices of the figure after a translation 6 units up and 2 units right?

What are the vertices of the figure after a translation 2 units right, followed by a reflection in the  $x$ -axis?

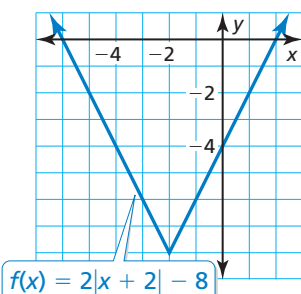
What are the vertices of the figure after a translation 6 units up, followed by a reflection in the  $x$ -axis?



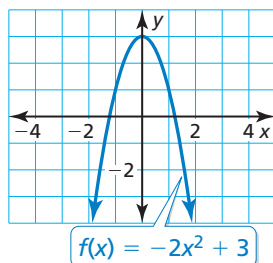
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the function family to which  $f$  belongs. Compare the graph of  $f$  to the graph of its parent function. (See Example 1.)

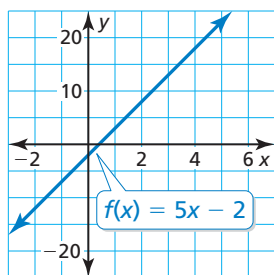
3.



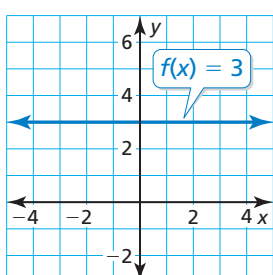
4.



5.



6.



- MODELING WITH MATHEMATICS** At 8:00 A.M., the temperature is  $43^\circ\text{F}$ . The temperature increases  $2^\circ\text{F}$  each hour for the next 7 hours. Graph the temperatures over time  $t$  ( $t = 0$  represents 8:00 A.M.). What type of function can you use to model the data? Explain.
- MODELING WITH MATHEMATICS** You purchase a car from a dealership for \$10,000. The trade-in value of the car each year after the purchase is given by the function  $f(x) = 10,000 - 250x^2$ . What type of function models the trade-in value?

In Exercises 9–18, graph the function and its parent function. Then describe the transformation. (See Examples 2 and 3.)

- $g(x) = x + 4$
- $f(x) = x - 6$
- $f(x) = x^2 - 1$
- $h(x) = (x + 4)^2$
- $g(x) = |x - 5|$
- $f(x) = 4 + |x|$
- $h(x) = -x^2$
- $g(x) = -x$
- $f(x) = 3$
- $f(x) = -2$

In Exercises 19–26, graph the function and its parent function. Then describe the transformation. (See Example 4.)

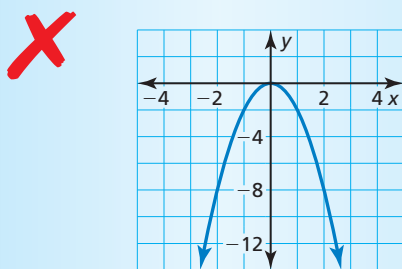
19.  $f(x) = \frac{1}{3}x$       20.  $g(x) = 4x$   
 21.  $f(x) = 2x^2$       22.  $h(x) = \frac{1}{3}x^2$   
 23.  $h(x) = \frac{3}{4}x$       24.  $g(x) = \frac{4}{3}x$   
 25.  $h(x) = 3|x|$       26.  $f(x) = \frac{1}{2}|x|$

In Exercises 27–34, use a graphing calculator to graph the function and its parent function. Then describe the transformations. (See Example 5.)

27.  $f(x) = 3x + 2$       28.  $h(x) = -x + 5$   
 29.  $h(x) = -3|x| - 1$       30.  $f(x) = \frac{3}{4}|x| + 1$   
 31.  $g(x) = \frac{1}{2}x^2 - 6$       32.  $f(x) = 4x^2 - 3$   
 33.  $f(x) = -(x + 3)^2 + \frac{1}{4}$   
 34.  $g(x) = -|x - 1| - \frac{1}{2}$

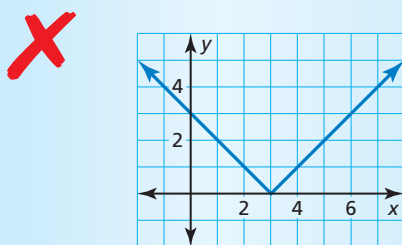
**ERROR ANALYSIS** In Exercises 35 and 36, identify and correct the error in describing the transformation of the parent function.

35.



The graph is a reflection in the  $x$ -axis and a vertical shrink of the parent quadratic function.

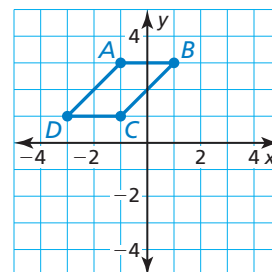
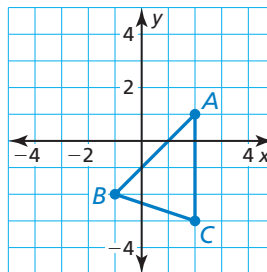
36.



The graph is a translation 3 units right of the parent absolute value function, so the function is  $f(x) = |x + 3|$ .

**MATHEMATICAL CONNECTIONS** In Exercises 37 and 38, find the coordinates of the figure after the transformation.

37. Translate 2 units down.      38. Reflect in the  $x$ -axis.



**USING TOOLS** In Exercises 39–44, identify the function family and describe the domain and range. Use a graphing calculator to verify your answer.

39.  $g(x) = |x + 2| - 1$       40.  $h(x) = |x - 3| + 2$   
 41.  $g(x) = 3x + 4$       42.  $f(x) = -4x + 11$   
 43.  $f(x) = 5x^2 - 2$       44.  $f(x) = -2x^2 + 6$

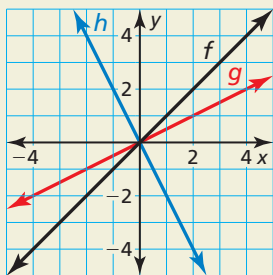
45. **MODELING WITH MATHEMATICS** The table shows the speeds of a car as it travels through an intersection with a stop sign. What type of function can you use to model the data? Estimate the speed of the car when it is 20 yards past the intersection. (See Example 6.)

Displacement from sign (yards), $x$	Speed (miles per hour), $y$
-100	40
-50	20
-10	4
0	0
10	4
50	20
100	40

46. **THOUGHT PROVOKING** In the same coordinate plane, sketch the graph of the parent quadratic function and the graph of a quadratic function that has no  $x$ -intercepts. Describe the transformation(s) of the parent function.

47. **USING STRUCTURE** Graph the functions  $f(x) = |x - 4|$  and  $g(x) = |x| - 4$ . Are they equivalent? Explain.

48. **HOW DO YOU SEE IT?** Consider the graphs of  $f$ ,  $g$ , and  $h$ .



- a. Does the graph of  $g$  represent a vertical stretch or a vertical shrink of the graph of  $f$ ? Explain your reasoning.
- b. Describe how to transform the graph of  $f$  to obtain the graph of  $h$ .
49. **MAKING AN ARGUMENT** Your friend says two different translations of the graph of the parent linear function can result in the graph of  $f(x) = x - 2$ . Is your friend correct? Explain.
50. **DRAWING CONCLUSIONS** A person swims at a constant speed of 1 meter per second. What type of function can be used to model the distance the swimmer travels? If the person has a 10-meter head start, what type of transformation does this represent? Explain.



51. **PROBLEM SOLVING** You are playing basketball with your friends. The height (in feet) of the ball above the ground  $t$  seconds after a shot is released from your hand is modeled by the function  $f(t) = -16t^2 + 32t + 5.2$ .
- a. Without graphing, identify the type of function that models the height of the basketball.
- b. What is the value of  $t$  when the ball is released from your hand? Explain your reasoning.
- c. How many feet above the ground is the ball when it is released from your hand? Explain.
52. **MODELING WITH MATHEMATICS** The table shows the battery lives of a computer over time. What type of function can you use to model the data? Interpret the meaning of the  $x$ -intercept in this situation.

Time (hours), $x$	Battery life remaining, $y$
1	80%
3	40%
5	0%
6	20%
8	60%

53. **REASONING** Compare each function with its parent function. State whether it contains a *horizontal translation*, *vertical translation*, *both*, or *neither*. Explain your reasoning.
- a.  $f(x) = 2|x| - 3$       b.  $f(x) = (x - 8)^2$
- c.  $f(x) = |x + 2| + 4$       d.  $f(x) = 4x^2$
54. **CRITICAL THINKING** Use the values  $-1$ ,  $0$ ,  $1$ , and  $2$  in the correct box so the graph of each function intersects the  $x$ -axis. Explain your reasoning.
- a.  $f(x) = 3x \square + 1$       b.  $f(x) = |2x - 6| - \square$
- c.  $f(x) = \square x^2 + 1$       d.  $f(x) = \square$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the ordered pair is a solution of the equation. (*Skills Review Handbook*)

55.  $f(x) = |x + 2|$ ;  $(1, -3)$       56.  $f(x) = |x| - 3$ ;  $(-2, -5)$
57.  $f(x) = x - 3$ ;  $(5, 2)$       58.  $f(x) = x - 4$ ;  $(12, 8)$

Find the  $x$ -intercept and the  $y$ -intercept of the graph of the equation. (*Skills Review Handbook*)

59.  $y = x$       60.  $y = x + 2$
61.  $3x + y = 1$       62.  $x - 2y = 8$

# 1.2 Transformations of Linear and Absolute Value Functions

**Essential Question** How do the graphs of  $y = f(x) + k$ ,  $y = f(x - h)$ , and  $y = -f(x)$  compare to the graph of the parent function  $f$ ?

## USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technological tools to visualize results and explore consequences.

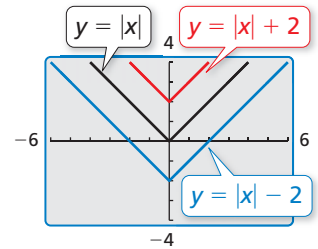
### EXPLORATION 1 Transformations of the Parent Absolute Value Function

**Work with a partner.** Compare the graph of the function

$$y = |x| + k \quad \text{Transformation}$$

to the graph of the parent function

$$f(x) = |x|. \quad \text{Parent function}$$



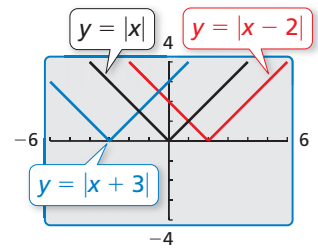
### EXPLORATION 2 Transformations of the Parent Absolute Value Function

**Work with a partner.** Compare the graph of the function

$$y = |x - h| \quad \text{Transformation}$$

to the graph of the parent function

$$f(x) = |x|. \quad \text{Parent function}$$



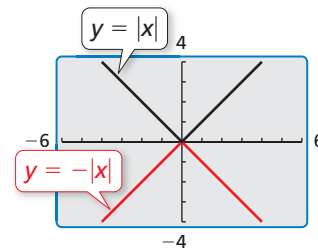
### EXPLORATION 3 Transformation of the Parent Absolute Value Function

**Work with a partner.** Compare the graph of the function

$$y = -|x| \quad \text{Transformation}$$

to the graph of the parent function

$$f(x) = |x|. \quad \text{Parent function}$$



## Communicate Your Answer

- How do the graphs of  $y = f(x) + k$ ,  $y = f(x - h)$ , and  $y = -f(x)$  compare to the graph of the parent function  $f$ ?
- Compare the graph of each function to the graph of its parent function  $f$ . Use a graphing calculator to verify your answers are correct.
 

a. $y = \sqrt{x} - 4$	b. $y = \sqrt{x + 4}$	c. $y = -\sqrt{x}$
d. $y = x^2 + 1$	e. $y = (x - 1)^2$	f. $y = -x^2$

# 1.2 Lesson

## What You Will Learn

- ▶ Write functions representing translations and reflections.
- ▶ Write functions representing stretches and shrinks.
- ▶ Write functions representing combinations of transformations.

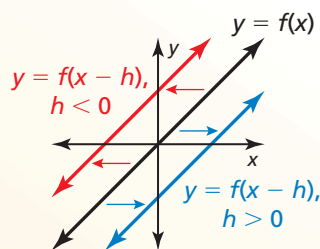
## Translations and Reflections

You can use function notation to represent transformations of graphs of functions.

### Core Concept

#### Horizontal Translations

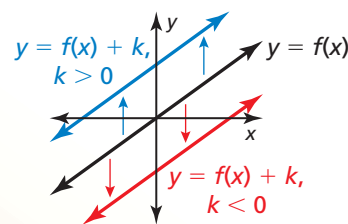
The graph of  $y = f(x - h)$  is a horizontal translation of the graph of  $y = f(x)$ , where  $h \neq 0$ .



Subtracting  $h$  from the **inputs** before evaluating the function shifts the graph left when  $h < 0$  and right when  $h > 0$ .

#### Vertical Translations

The graph of  $y = f(x) + k$  is a vertical translation of the graph of  $y = f(x)$ , where  $k \neq 0$ .



Adding  $k$  to the **outputs** shifts the graph down when  $k < 0$  and up when  $k > 0$ .

### EXAMPLE 1 Writing Translations of Functions

Let  $f(x) = 2x + 1$ .

- a. Write a function  $g$  whose graph is a translation 3 units down of the graph of  $f$ .
- b. Write a function  $h$  whose graph is a translation 2 units to the left of the graph of  $f$ .

#### SOLUTION

- a. A translation 3 units down is a vertical translation that adds  $-3$  to each output value.

$$\begin{aligned} g(x) &= f(x) + (-3) && \text{Add } -3 \text{ to the output.} \\ &= 2x + 1 + (-3) && \text{Substitute } 2x + 1 \text{ for } f(x). \\ &= 2x - 2 && \text{Simplify.} \end{aligned}$$

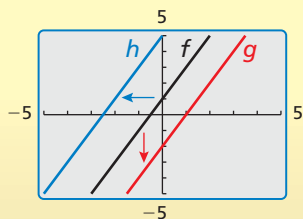
- ▶ The translated function is  $g(x) = 2x - 2$ .

- b. A translation 2 units to the left is a horizontal translation that subtracts  $-2$  from each input value.

$$\begin{aligned} h(x) &= f(x - (-2)) && \text{Subtract } -2 \text{ from the input.} \\ &= f(x + 2) && \text{Add the opposite.} \\ &= 2(x + 2) + 1 && \text{Replace } x \text{ with } x + 2 \text{ in } f(x). \\ &= 2x + 5 && \text{Simplify.} \end{aligned}$$

- ▶ The translated function is  $h(x) = 2x + 5$ .

#### Check



## Core Concept

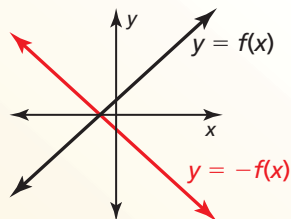
### STUDY TIP

When you reflect a function in a line, the graphs are symmetric about that line.



### Reflections in the $x$ -axis

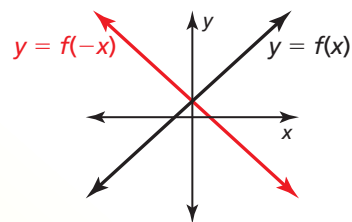
The graph of  $y = -f(x)$  is a reflection in the  $x$ -axis of the graph of  $y = f(x)$ .



Multiplying the **outputs** by  $-1$  changes their signs.

### Reflections in the $y$ -axis

The graph of  $y = f(-x)$  is a reflection in the  $y$ -axis of the graph of  $y = f(x)$ .



Multiplying the **inputs** by  $-1$  changes their signs.

### EXAMPLE 2 Writing Reflections of Functions

Let  $f(x) = |x + 3| + 1$ .

- Write a function  $g$  whose graph is a reflection in the  $x$ -axis of the graph of  $f$ .
- Write a function  $h$  whose graph is a reflection in the  $y$ -axis of the graph of  $f$ .

#### SOLUTION

- A reflection in the  $x$ -axis changes the sign of each output value.

$$\begin{aligned} g(x) &= -f(x) && \text{Multiply the output by } -1. \\ &= -(|x + 3| + 1) && \text{Substitute } |x + 3| + 1 \text{ for } f(x). \\ &= -|x + 3| - 1 && \text{Distributive Property} \end{aligned}$$

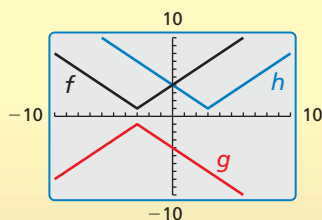
- ▶ The reflected function is  $g(x) = -|x + 3| - 1$ .

- A reflection in the  $y$ -axis changes the sign of each input value.

$$\begin{aligned} h(x) &= f(-x) && \text{Multiply the input by } -1. \\ &= |-x + 3| + 1 && \text{Replace } x \text{ with } -x \text{ in } f(x). \\ &= |-(x - 3)| + 1 && \text{Factor out } -1. \\ &= |-1| \cdot |x - 3| + 1 && \text{Product Property of Absolute Value} \\ &= |x - 3| + 1 && \text{Simplify.} \end{aligned}$$

- ▶ The reflected function is  $h(x) = |x - 3| + 1$ .

#### Check



### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . Use a graphing calculator to check your answer.

- $f(x) = 3x$ ; translation 5 units up
- $f(x) = |x| - 3$ ; translation 4 units to the right
- $f(x) = -|x + 2| - 1$ ; reflection in the  $x$ -axis
- $f(x) = \frac{1}{2}x + 1$ ; reflection in the  $y$ -axis



## Stretches and Shrinks

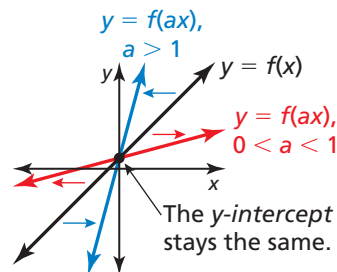
In the previous section, you learned that vertical stretches and shrinks transform graphs. You can also use *horizontal* stretches and shrinks to transform graphs.

### Core Concept

#### Horizontal Stretches and Shrinks

The graph of  $y = f(ax)$  is a horizontal stretch or shrink by a factor of  $\frac{1}{a}$  of the graph of  $y = f(x)$ , where  $a > 0$  and  $a \neq 1$ .

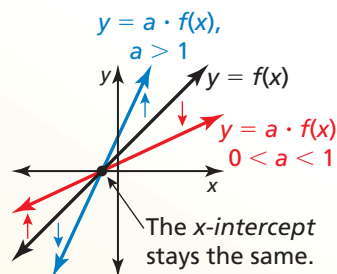
Multiplying the **inputs** by  $a$  before evaluating the function stretches the graph horizontally (away from the  $y$ -axis) when  $0 < a < 1$ , and shrinks the graph horizontally (toward the  $y$ -axis) when  $a > 1$ .



#### Vertical Stretches and Shrinks

The graph of  $y = a \cdot f(x)$  is a vertical stretch or shrink by a factor of  $a$  of the graph of  $y = f(x)$ , where  $a > 0$  and  $a \neq 1$ .

Multiplying the **outputs** by  $a$  stretches the graph vertically (away from the  $x$ -axis) when  $a > 1$ , and shrinks the graph vertically (toward the  $x$ -axis) when  $0 < a < 1$ .



#### STUDY TIP

The graphs of  $y = f(-ax)$  and  $y = -a \cdot f(x)$  represent a stretch or shrink *and* a reflection in the  $x$ - or  $y$ -axis of the graph of  $y = f(x)$ .



### EXAMPLE 3 Writing Stretches and Shrinks of Functions

Let  $f(x) = |x - 3| - 5$ . Write (a) a function  $g$  whose graph is a horizontal shrink of the graph of  $f$  by a factor of  $\frac{1}{3}$ , and (b) a function  $h$  whose graph is a vertical stretch of the graph of  $f$  by a factor of 2.

#### SOLUTION

- a. A horizontal shrink by a factor of  $\frac{1}{3}$  multiplies each input value by 3.

$$\begin{aligned} g(x) &= f(3x) && \text{Multiply the input by 3.} \\ &= |3x - 3| - 5 && \text{Replace } x \text{ with } 3x \text{ in } f(x). \end{aligned}$$

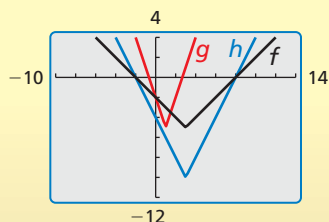
- ▶ The transformed function is  $g(x) = |3x - 3| - 5$ .

- b. A vertical stretch by a factor of 2 multiplies each output value by 2.

$$\begin{aligned} h(x) &= 2 \cdot f(x) && \text{Multiply the output by 2.} \\ &= 2 \cdot (|x - 3| - 5) && \text{Substitute } |x - 3| - 5 \text{ for } f(x). \\ &= 2|x - 3| - 10 && \text{Distributive Property} \end{aligned}$$

- ▶ The transformed function is  $h(x) = 2|x - 3| - 10$ .

#### Check



### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . Use a graphing calculator to check your answer.

- $f(x) = 4x + 2$ ; horizontal stretch by a factor of 2
- $f(x) = |x| - 3$ ; vertical shrink by a factor of  $\frac{1}{3}$

## Combinations of Transformations

You can write a function that represents a series of transformations on the graph of another function by applying the transformations one at a time in the stated order.

### EXAMPLE 4 Combining Transformations

Let the graph of  $g$  be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of  $f(x) = x$ . Write a rule for  $g$ .

#### SOLUTION

**Step 1** First write a function  $h$  that represents the vertical shrink of  $f$ .

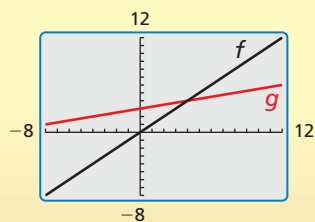
$$\begin{aligned} h(x) &= 0.25 \cdot f(x) && \text{Multiply the output by 0.25.} \\ &= 0.25x && \text{Substitute } x \text{ for } f(x). \end{aligned}$$

**Step 2** Then write a function  $g$  that represents the translation of  $h$ .

$$\begin{aligned} g(x) &= h(x) + 3 && \text{Add 3 to the output.} \\ &= 0.25x + 3 && \text{Substitute } 0.25x \text{ for } h(x). \end{aligned}$$

► The transformed function is  $g(x) = 0.25x + 3$ .

#### Check



### EXAMPLE 5 Modeling with Mathematics

You design a computer game. Your revenue for  $x$  downloads is given by  $f(x) = 2x$ . Your profit is \$50 less than 90% of the revenue for  $x$  downloads. Describe how to transform the graph of  $f$  to model the profit. What is your profit for 100 downloads?

#### SOLUTION

- 1. Understand the Problem** You are given a function that represents your revenue and a verbal statement that represents your profit. You are asked to find the profit for 100 downloads.
- 2. Make a Plan** Write a function  $p$  that represents your profit. Then use this function to find the profit for 100 downloads.
- 3. Solve the Problem** profit = 90% • revenue – 50

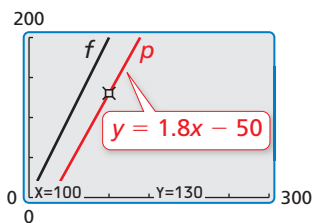
$$\begin{aligned} p(x) &= 0.9 \cdot f(x) - 50 && \begin{array}{l} \text{Vertical shrink by a factor of 0.9} \\ \text{Translation 50 units down} \end{array} \\ &= 0.9 \cdot 2x - 50 && \text{Substitute } 2x \text{ for } f(x). \\ &= 1.8x - 50 && \text{Simplify.} \end{aligned}$$

To find the profit for 100 downloads, evaluate  $p$  when  $x = 100$ .

$$p(100) = 1.8(100) - 50 = 130$$

► Your profit is \$130 for 100 downloads.

- 4. Look Back** The vertical shrink decreases the slope, and the translation shifts the graph 50 units down. So, the graph of  $p$  is below and not as steep as the graph of  $f$ .



### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- Let the graph of  $g$  be a translation 6 units down followed by a reflection in the  $x$ -axis of the graph of  $f(x) = |x|$ . Write a rule for  $g$ . Use a graphing calculator to check your answer.
- WHAT IF?** In Example 5, your revenue function is  $f(x) = 3x$ . How does this affect your profit for 100 downloads?

## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** The function  $g(x) = |5x| - 4$  is a horizontal \_\_\_\_\_ of the function  $f(x) = |x| - 4$ .
- WHICH ONE DOESN'T BELONG?** Which transformation does *not* belong with the other three? Explain your reasoning.

Translate the graph of  $f(x) = 2x + 3$  up 2 units.

Shrink the graph of  $f(x) = x + 5$  horizontally by a factor of  $\frac{1}{2}$ .

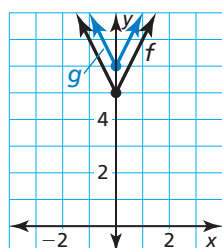
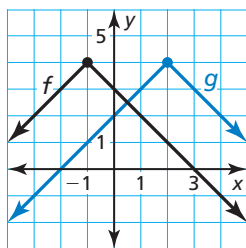
Stretch the graph of  $f(x) = x + 3$  vertically by a factor of 2.

Translate the graph of  $f(x) = 2x + 3$  left 1 unit.

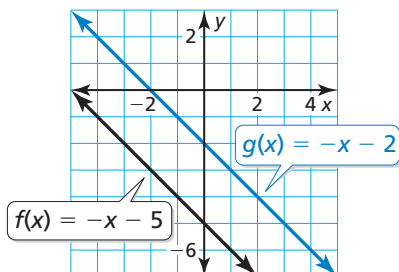
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . Use a graphing calculator to check your answer. (See Example 1.)

- $f(x) = x - 5$ ; translation 4 units to the left
- $f(x) = x + 2$ ; translation 2 units to the right
- $f(x) = |4x + 3| + 2$ ; translation 2 units down
- $f(x) = 2x - 9$ ; translation 6 units up
- $f(x) = 4 - |x + 1|$       8.  $f(x) = |4x| + 5$



- WRITING** Describe two different translations of the graph of  $f$  that result in the graph of  $g$ .



- PROBLEM SOLVING** You open a café. The function  $f(x) = 4000x$  represents your expected net income (in dollars) after being open  $x$  weeks. Before you open, you incur an extra expense of \$12,000. What transformation of  $f$  is necessary to model this situation? How many weeks will it take to pay off the extra expense?

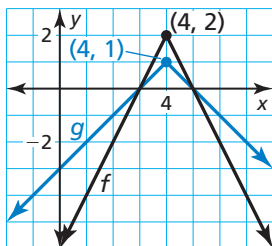


In Exercises 11–16, write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . Use a graphing calculator to check your answer. (See Example 2.)

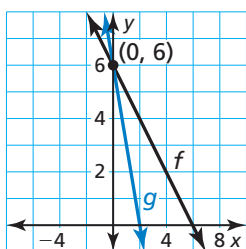
- $f(x) = -5x + 2$ ; reflection in the  $x$ -axis
- $f(x) = \frac{1}{2}x - 3$ ; reflection in the  $x$ -axis
- $f(x) = |6x| - 2$ ; reflection in the  $y$ -axis
- $f(x) = |2x - 1| + 3$ ; reflection in the  $y$ -axis
- $f(x) = -3 + |x - 11|$ ; reflection in the  $y$ -axis
- $f(x) = -x + 1$ ; reflection in the  $y$ -axis

In Exercises 17–22, write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . Use a graphing calculator to check your answer. (See Example 3.)

17.  $f(x) = x + 2$ ; vertical stretch by a factor of 5
18.  $f(x) = 2x + 6$ ; vertical shrink by a factor of  $\frac{1}{2}$
19.  $f(x) = |2x| + 4$ ; horizontal shrink by a factor of  $\frac{1}{2}$
20.  $f(x) = |x + 3|$ ; horizontal stretch by a factor of 4
21.  $f(x) = -2|x - 4| + 2$

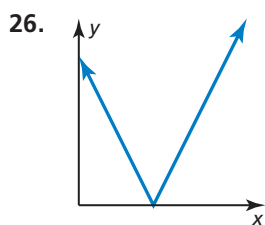
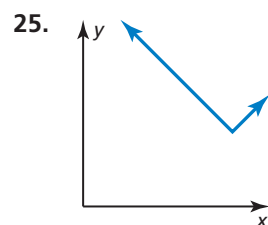
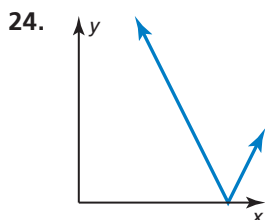
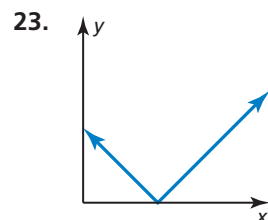
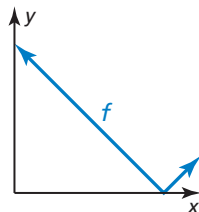


22.  $f(x) = 6 - x$



### ANALYZING RELATIONSHIPS

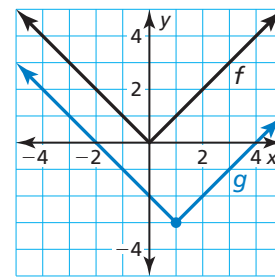
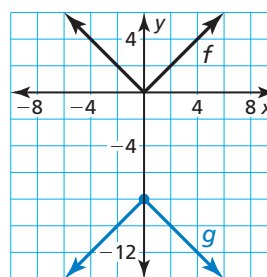
In Exercises 23–26, match the graph of the transformation of  $f$  with the correct equation shown. Explain your reasoning.



- A.  $y = 2f(x)$       B.  $y = f(2x)$   
 C.  $y = f(x + 2)$       D.  $y = f(x) + 2$

In Exercises 27–32, write a function  $g$  whose graph represents the indicated transformations of the graph of  $f$ . (See Example 4.)

27.  $f(x) = x$ ; vertical stretch by a factor of 2 followed by a translation 1 unit up
28.  $f(x) = x$ ; translation 3 units down followed by a vertical shrink by a factor of  $\frac{1}{3}$
29.  $f(x) = |x|$ ; translation 2 units to the right followed by a horizontal stretch by a factor of 2
30.  $f(x) = |x|$ ; reflection in the  $y$ -axis followed by a translation 3 units to the right
31.  $f(x) = |x|$       32.  $f(x) = |x|$



**ERROR ANALYSIS** In Exercises 33 and 34, identify and correct the error in writing the function  $g$  whose graph represents the indicated transformations of the graph of  $f$ .

33.  $f(x) = |x|$ ; translation 3 units to the right followed by a translation 2 units up
- $g(x) = |x + 3| + 2$

34.  $f(x) = x$ ; translation 6 units down followed by a vertical stretch by a factor of 5
- $g(x) = 5x - 6$

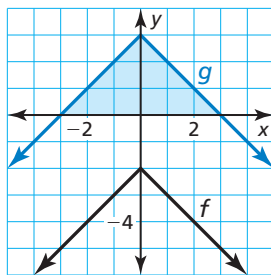
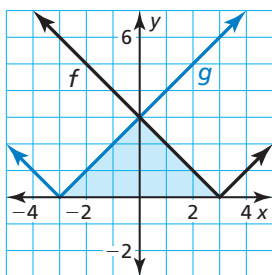
35. **MAKING AN ARGUMENT** Your friend claims that when writing a function whose graph represents a combination of transformations, the order is not important. Is your friend correct? Justify your answer.

- 36. MODELING WITH MATHEMATICS** During a recent period of time, bookstore sales have been declining. The sales (in billions of dollars) can be modeled by the function  $f(t) = -\frac{7}{5}t + 17.2$ , where  $t$  is the number of years since 2006. Suppose sales decreased at twice the rate. How can you transform the graph of  $f$  to model the sales? Explain how the sales in 2010 are affected by this change. (See Example 5.)

**MATHEMATICAL CONNECTIONS** For Exercises 37–40, describe the transformation of the graph of  $f$  to the graph of  $g$ . Then find the area of the shaded triangle.

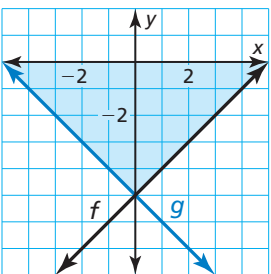
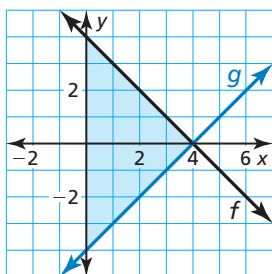
37.  $f(x) = |x - 3|$

38.  $f(x) = -|x| - 2$



39.  $f(x) = -x + 4$

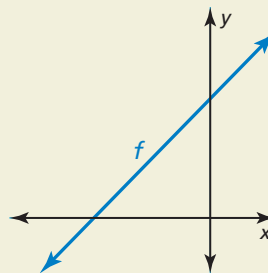
40.  $f(x) = x - 5$



- 41. ABSTRACT REASONING** The functions  $f(x) = mx + b$  and  $g(x) = mx + c$  represent two parallel lines.

- Write an expression for the vertical translation of the graph of  $f$  to the graph of  $g$ .
- Use the definition of slope to write an expression for the horizontal translation of the graph of  $f$  to the graph of  $g$ .

- 42. HOW DO YOU SEE IT?** Consider the graph of  $f(x) = mx + b$ . Describe the effect each transformation has on the slope of the line and the intercepts of the graph.



- Reflect the graph of  $f$  in the  $y$ -axis.
- Shrink the graph of  $f$  vertically by a factor of  $\frac{1}{3}$ .
- Stretch the graph of  $f$  horizontally by a factor of 2.

- 43. REASONING** The graph of  $g(x) = -4|x| + 2$  is a reflection in the  $x$ -axis, vertical stretch by a factor of 4, and a translation 2 units down of the graph of its parent function. Choose the correct order for the transformations of the graph of the parent function to obtain the graph of  $g$ . Explain your reasoning.

- 44. THOUGHT PROVOKING** You are planning a cross-country bicycle trip of 4320 miles. Your distance  $d$  (in miles) from the halfway point can be modeled by  $d = 72|x - 30|$ , where  $x$  is the time (in days) and  $x = 0$  represents June 1. Your plans are altered so that the model is now a right shift of the original model. Give an example of how this can happen. Sketch both the original model and the shifted model.

- 45. CRITICAL THINKING** Use the correct value 0,  $-2$ , or 1 with  $a$ ,  $b$ , and  $c$  so the graph of  $g(x) = a|x - b| + c$  is a reflection in the  $x$ -axis followed by a translation one unit to the left and one unit up of the graph of  $f(x) = 2|x - 2| + 1$ . Explain your reasoning.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the function for the given value of  $x$ . (Skills Review Handbook)

46.  $f(x) = x + 4$ ;  $x = 3$

47.  $f(x) = 4x - 1$ ;  $x = -1$

48.  $f(x) = -x + 3$ ;  $x = 5$

49.  $f(x) = -2x - 2$ ;  $x = -1$

Create a scatter plot of the data. (Skills Review Handbook)

50.

$x$	8	10	11	12	15
$f(x)$	4	9	10	12	12

51.

$x$	2	5	6	10	13
$f(x)$	22	13	15	12	6

# 1.1–1.2 What Did You Learn?

## Core Vocabulary

parent function, *p. 4*  
transformation, *p. 5*  
translation, *p. 5*

reflection, *p. 5*  
vertical stretch, *p. 6*  
vertical shrink, *p. 6*

## Core Concepts

### Section 1.1

Parent Functions, *p. 4*

Describing Transformations, *p. 5*

### Section 1.2

Horizontal Translations, *p. 12*  
Vertical Translations, *p. 12*  
Reflections in the  $x$ -axis, *p. 13*

Reflections in the  $y$ -axis, *p. 13*  
Horizontal Stretches and Shrinks, *p. 14*  
Vertical Stretches and Shrinks, *p. 14*

## Mathematical Practices

1. How can you analyze the values given in the table in Exercise 45 on page 9 to help you determine what type of function models the data?
2. Explain how you would round your answer in Exercise 10 on page 16 if the extra expense is \$13,500.

## Study Skills

# Taking Control of Your Class Time

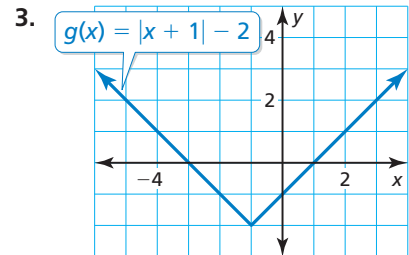
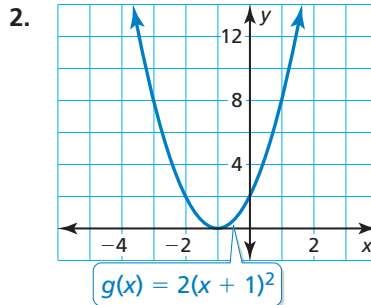
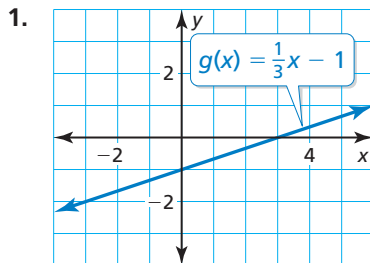
1. Sit where you can easily see and hear the teacher, and the teacher can see you.
2. Pay attention to what the teacher says about math, not just what is written on the board.
3. Ask a question if the teacher is moving through the material too fast.
4. Try to memorize new information while learning it.
5. Ask for clarification if you do not understand something.
6. Think as intensely as if you were going to take a quiz on the material at the end of class.
7. Volunteer when the teacher asks for someone to go up to the board.
8. At the end of class, identify concepts or problems for which you still need clarification.
9. Use the tutorials at [BigIdeasMath.com](http://BigIdeasMath.com) for additional help.





# 1.1–1.2 Quiz

Identify the function family to which  $g$  belongs. Compare the graph of the function to the graph of its parent function. (Section 1.1)



Graph the function and its parent function. Then describe the transformation. (Section 1.1)

4.  $f(x) = \frac{3}{2}$

5.  $f(x) = 3x$

6.  $f(x) = 2(x - 1)^2$

7.  $f(x) = -|x + 2| - 7$

8.  $f(x) = \frac{1}{4}x^2 + 1$

9.  $f(x) = -\frac{1}{2}x - 4$

Write a function  $g$  whose graph represents the indicated transformation of the graph of  $f$ . (Section 1.2)

10.  $f(x) = 2x + 1$ ; translation 3 units up

11.  $f(x) = -3|x - 4|$ ; vertical shrink by a factor of  $\frac{1}{2}$

12.  $f(x) = 3|x + 5|$ ; reflection in the  $x$ -axis

13.  $f(x) = \frac{1}{3}x - \frac{2}{3}$ ; translation 4 units left

Write a function  $g$  whose graph represents the indicated transformations of the graph of  $f$ . (Section 1.2)

14. Let  $g$  be a translation 2 units down and a horizontal shrink by a factor of  $\frac{2}{3}$  of the graph of  $f(x) = x$ .

15. Let  $g$  be a translation 9 units down followed by a reflection in the  $y$ -axis of the graph of  $f(x) = x$ .

16. Let  $g$  be a reflection in the  $x$ -axis and a vertical stretch by a factor of 4 followed by a translation 7 units down and 1 unit right of the graph of  $f(x) = |x|$ .

17. Let  $g$  be a translation 1 unit down and 2 units left followed by a vertical shrink by a factor of  $\frac{1}{2}$  of the graph of  $f(x) = |x|$ .

18. The table shows the total distance a new car travels each month after it is purchased. What type of function can you use to model the data? Estimate the mileage after 1 year. (Section 1.1)

Time (months), $x$	0	2	5	6	9
Distance (miles), $y$	0	2300	5750	6900	10,350

19. The total cost of an annual pass plus camping for  $x$  days in a National Park can be modeled by the function  $f(x) = 20x + 80$ . Senior citizens pay half of this price and receive an additional \$30 discount. Describe how to transform the graph of  $f$  to model the total cost for a senior citizen. What is the total cost for a senior citizen to go camping for three days? (Section 1.2)

# 1.3 Modeling with Linear Functions

**Essential Question** How can you use a linear function to model and analyze a real-life situation?

## EXPLORATION 1 Modeling with a Linear Function

**Work with a partner.** A company purchases a copier for \$12,000. The spreadsheet shows how the copier depreciates over an 8-year period.

	A	B
1	Year, $t$	Value, $V$
2	0	\$12,000
3	1	\$10,750
4	2	\$9,500
5	3	\$8,250
6	4	\$7,000
7	5	\$5,750
8	6	\$4,500
9	7	\$3,250
10	8	\$2,000
11		

- Write a linear function to represent the value  $V$  of the copier as a function of the number  $t$  of years.
- Sketch a graph of the function. Explain why this type of depreciation is called *straight line depreciation*.
- Interpret the slope of the graph in the context of the problem.

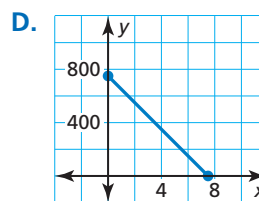
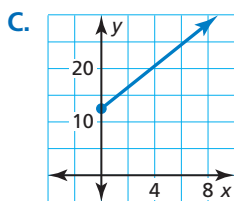
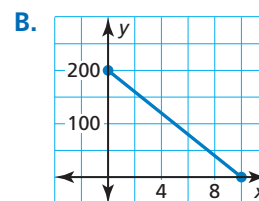
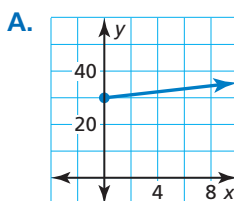
### MODELING WITH MATHEMATICS

To be proficient in math, you need to routinely interpret your results in the context of the situation.

## EXPLORATION 2 Modeling with Linear Functions

**Work with a partner.** Match each description of the situation with its corresponding graph. Explain your reasoning.

- A person gives \$20 per week to a friend to repay a \$200 loan.
- An employee receives \$12.50 per hour plus \$2 for each unit produced per hour.
- A sales representative receives \$30 per day for food plus \$0.565 for each mile driven.
- A computer that was purchased for \$750 depreciates \$100 per year.



### Communicate Your Answer

- How can you use a linear function to model and analyze a real-life situation?
- Use the Internet or some other reference to find a real-life example of straight line depreciation.
  - Use a spreadsheet to show the depreciation.
  - Write a function that models the depreciation.
  - Sketch a graph of the function.

# 1.3 Lesson

## Core Vocabulary

line of fit, p. 24  
 line of best fit, p. 25  
 correlation coefficient, p. 25

### Previous

slope  
 slope-intercept form  
 point-slope form  
 scatter plot

## What You Will Learn

- ▶ Write equations of linear functions using points and slopes.
- ▶ Find lines of fit and lines of best fit.

## Writing Linear Equations

### Core Concept

#### Writing an Equation of a Line

**Given slope  $m$  and  $y$ -intercept  $b$**

Use slope-intercept form:

$$y = mx + b$$

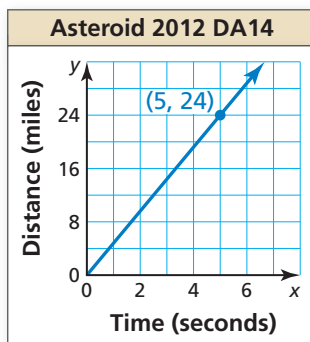
**Given slope  $m$  and a point  $(x_1, y_1)$**

Use point-slope form:

$$y - y_1 = m(x - x_1)$$

**Given points  $(x_1, y_1)$  and  $(x_2, y_2)$**

First use the slope formula to find  $m$ .  
 Then use point-slope form with either given point.



### EXAMPLE 1 Writing a Linear Equation from a Graph

The graph shows the distance Asteroid 2012 DA14 travels in  $x$  seconds. Write an equation of the line and interpret the slope. The asteroid came within 17,200 miles of Earth in February, 2013. About how long does it take the asteroid to travel that distance?

#### SOLUTION

From the graph, you can see the slope is  $m = \frac{24}{5} = 4.8$  and the  $y$ -intercept is  $b = 0$ . Use slope-intercept form to write an equation of the line.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ &= 4.8x + 0 && \text{Substitute 4.8 for } m \text{ and 0 for } b. \end{aligned}$$

The equation is  $y = 4.8x$ . The slope indicates that the asteroid travels 4.8 miles per second. Use the equation to find how long it takes the asteroid to travel 17,200 miles.

$$\begin{aligned} 17,200 &= 4.8x && \text{Substitute 17,200 for } y. \\ 3583 &\approx x && \text{Divide each side by 4.8.} \end{aligned}$$

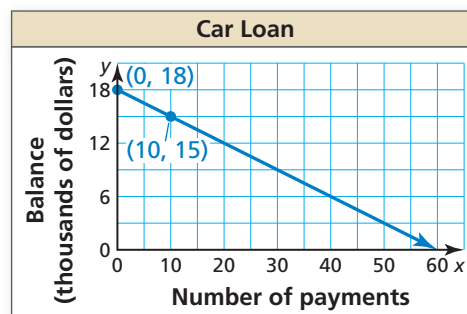
- ▶ Because there are 3600 seconds in 1 hour, it takes the asteroid about 1 hour to travel 17,200 miles.

### REMEMBER

An equation of the form  $y = mx$  indicates that  $x$  and  $y$  are in a proportional relationship.

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- The graph shows the remaining balance  $y$  on a car loan after making  $x$  monthly payments. Write an equation of the line and interpret the slope and  $y$ -intercept. What is the remaining balance after 36 payments?



**EXAMPLE 2** Modeling with Mathematics

Lakeside Inn	
Number of students, $x$	Total cost, $y$
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700



Two prom venues charge a rental fee plus a fee per student. The table shows the total costs for different numbers of students at Lakeside Inn. The total cost  $y$  (in dollars) for  $x$  students at Sunview Resort is represented by the equation

$$y = 10x + 600.$$

Which venue charges less per student? How many students must attend for the total costs to be the same?

**SOLUTION**

- 1. Understand the Problem** You are given an equation that represents the total cost at one venue and a table of values showing total costs at another venue. You need to compare the costs.
- 2. Make a Plan** Write an equation that models the total cost at Lakeside Inn. Then compare the slopes to determine which venue charges less per student. Finally, equate the cost expressions and solve to determine the number of students for which the total costs are equal.
- 3. Solve the Problem** First find the slope using any two points from the table. Use  $(x_1, y_1) = (100, 1500)$  and  $(x_2, y_2) = (125, 1800)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1500}{125 - 100} = \frac{300}{25} = 12$$

Write an equation that represents the total cost at Lakeside Inn using the slope of 12 and a point from the table. Use  $(x_1, y_1) = (100, 1500)$ .

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 1500 = 12(x - 100) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 1500 = 12x - 1200 \quad \text{Distributive Property}$$

$$y = 12x + 300 \quad \text{Add 1500 to each side.}$$

Equate the cost expressions and solve.

$$10x + 600 = 12x + 300 \quad \text{Set cost expressions equal.}$$

$$300 = 2x \quad \text{Combine like terms.}$$

$$150 = x \quad \text{Divide each side by 2.}$$

- Comparing the slopes of the equations, Sunview Resort charges \$10 per student, which is less than the \$12 per student that Lakeside Inn charges. The total costs are the same for 150 students.

- 4. Look Back** Notice that the table shows the total cost for 150 students at Lakeside Inn is \$2100. To check that your solution is correct, verify that the total cost at Sunview Resort is also \$2100 for 150 students.

$$y = 10(150) + 600 \quad \text{Substitute 150 for } x.$$

$$= 2100 \quad \checkmark \quad \text{Simplify.}$$

**Monitoring Progress**  Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- 2. WHAT IF?** Maple Ridge charges a rental fee plus a \$10 fee per student. The total cost is \$1900 for 140 students. Describe the number of students that must attend for the total cost at Maple Ridge to be less than the total costs at the other two venues. Use a graph to justify your answer.

## Finding Lines of Fit and Lines of Best Fit

Data do not always show an *exact* linear relationship. When the data in a scatter plot show an approximately linear relationship, you can model the data with a **line of fit**.

### Core Concept

#### Finding a Line of Fit

- Step 1** Create a scatter plot of the data.
- Step 2** Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.
- Step 3** Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.
- Step 4** Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

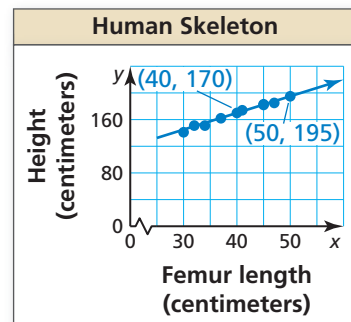
#### EXAMPLE 3 Finding a Line of Fit

Femur length, $x$	Height, $y$
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters long.

#### SOLUTION

- Step 1** Create a scatter plot of the data.  
The data show a linear relationship.
- Step 2** Sketch the line that most closely appears to fit the data. One possibility is shown.
- Step 3** Choose two points on the line.  
For the line shown, you might choose (40, 170) and (50, 195).
- Step 4** Write an equation of the line.



First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{195 - 170}{50 - 40} = \frac{25}{10} = 2.5$$

Use point-slope form to write an equation. Use  $(x_1, y_1) = (40, 170)$ .

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 170 = 2.5(x - 40) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 170 = 2.5x - 100 \quad \text{Distributive Property}$$

$$y = 2.5x + 70 \quad \text{Add 170 to each side.}$$

Use the equation to estimate the height of the person.

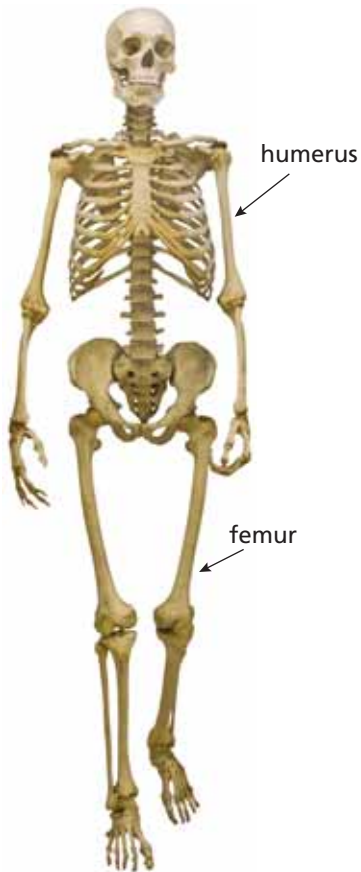
$$y = 2.5(35) + 70 \quad \text{Substitute 35 for } x.$$

$$= 157.5 \quad \text{Simplify.}$$

- The approximate height of a person with a 35-centimeter femur is 157.5 centimeters.

The **line of best fit** is the line that lies as close as possible to all of the data points. Many technology tools have a *linear regression* feature that you can use to find the line of best fit for a set of data.

The **correlation coefficient**, denoted by  $r$ , is a number from  $-1$  to  $1$  that measures how well a line fits a set of data pairs  $(x, y)$ . When  $r$  is near  $1$ , the points lie close to a line with a positive slope. When  $r$  is near  $-1$ , the points lie close to a line with a negative slope. When  $r$  is near  $0$ , the points do not lie close to any line.



### EXAMPLE 4 Using a Graphing Calculator

Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data in Example 3. Estimate the height of a person whose femur is 35 centimeters long. Compare this height to your estimate in Example 3.

#### SOLUTION

**Step 1** Enter the data into two lists.

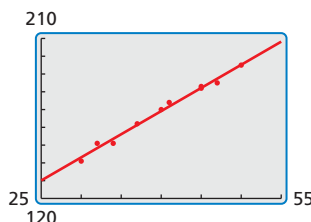
L1	L2	L3
40	170	---
45	183	---
32	151	---
50	195	---
37	162	---
41	174	---
30	141	---
L1(1)=40		

**Step 2** Use the *linear regression* feature. The line of best fit is  $y = 2.6x + 65$ .

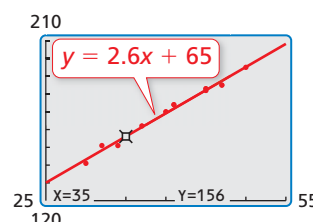
```
LinReg
y=ax+b
a=2.603570555
b=64.99682074
r^2=.9890669473
r=.9945184499
```

The value of  $r$  is close to 1.

**Step 3** Graph the regression equation with the scatter plot.



**Step 4** Use the *trace* feature to find the value of  $y$  when  $x = 35$ .



#### ATTENDING TO PRECISION

Be sure to analyze the data values to help you select an appropriate viewing window for your graph.

► The approximate height of a person with a 35-centimeter femur is 156 centimeters. This is less than the estimate found in Example 3.

#### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

3. The table shows the humerus lengths (in centimeters) and heights (in centimeters) of several females.

Humerus length, $x$	33	25	22	30	28	32	26	27
Height, $y$	166	142	130	154	152	159	141	145

- Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a female whose humerus is 40 centimeters long.
- Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data. Estimate the height of a female whose humerus is 40 centimeters long. Compare this height to your estimate in part (a).

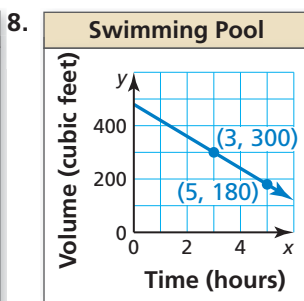
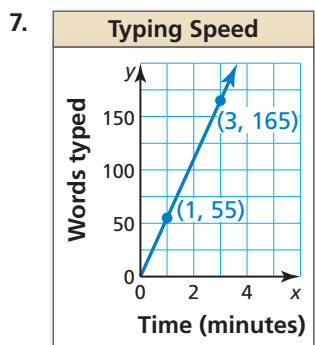
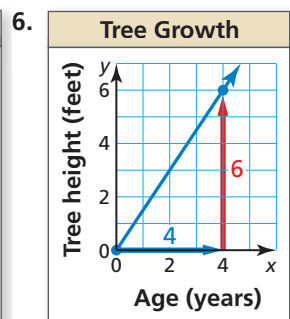
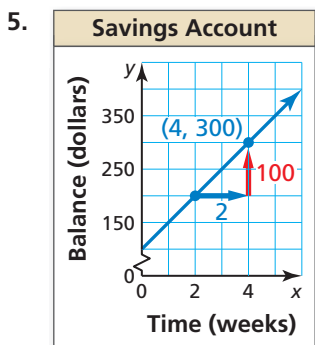
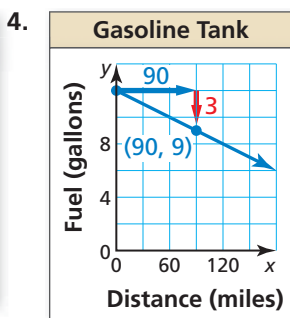
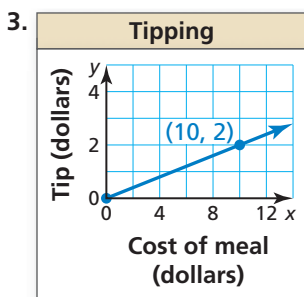


## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** The linear equation  $y = \frac{1}{2}x + 3$  is written in \_\_\_\_\_ form.
- VOCABULARY** A line of best fit has a correlation coefficient of  $-0.98$ . What can you conclude about the slope of the line?

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, use the graph to write an equation of the line and interpret the slope. (See Example 1.)



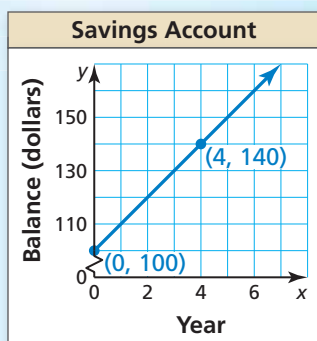
9. **MODELING WITH MATHEMATICS** Two newspapers charge a fee for placing an advertisement in their paper plus a fee based on the number of lines in the advertisement. The table shows the total costs for different length advertisements at the Daily Times. The total cost  $y$  (in dollars) for an advertisement that is  $x$  lines long at the Greenville Journal is represented by the equation  $y = 2x + 20$ . Which newspaper charges less per line? How many lines must be in an advertisement for the total costs to be the same? (See Example 2.)

Daily Times	
Number of lines, $x$	Total cost, $y$
4	27
5	30
6	33
7	36
8	39

10. **PROBLEM SOLVING** While on vacation in Canada, you notice that temperatures are reported in degrees Celsius. You know there is a linear relationship between Fahrenheit and Celsius, but you forget the formula. From science class, you remember the freezing point of water is  $0^\circ\text{C}$  or  $32^\circ\text{F}$ , and its boiling point is  $100^\circ\text{C}$  or  $212^\circ\text{F}$ .
- Write an equation that represents degrees Fahrenheit in terms of degrees Celsius.
  - The temperature outside is  $22^\circ\text{C}$ . What is this temperature in degrees Fahrenheit?
  - Rewrite your equation in part (a) to represent degrees Celsius in terms of degrees Fahrenheit.
  - The temperature of the hotel pool water is  $83^\circ\text{F}$ . What is this temperature in degrees Celsius?

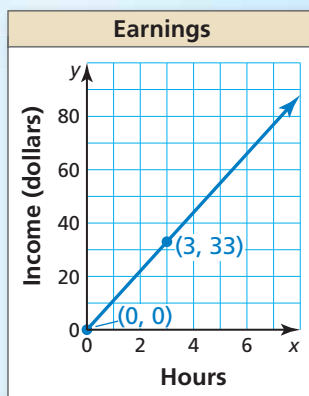
**ERROR ANALYSIS** In Exercises 11 and 12, describe and correct the error in interpreting the slope in the context of the situation.

11.



The slope of the line is 10, so after 7 years, the balance is \$70.

12.



The slope is 3, so the income is \$3 per hour.

In Exercises 13–16, determine whether the data show a linear relationship. If so, write an equation of a line of fit. Estimate  $y$  when  $x = 15$  and explain its meaning in the context of the situation. (See Example 3.)

13.	Minutes walking, $x$	1	6	11	13	16
	Calories burned, $y$	6	27	50	56	70

14.	Months, $x$	9	13	18	22	23
	Hair length (in.), $y$	3	5	7	10	11

15.	Hours, $x$	3	7	9	17	20
	Battery life (%), $y$	86	61	50	26	0

16.	Shoe size, $x$	6	8	8.5	10	13
	Heart rate (bpm), $y$	112	94	100	132	87

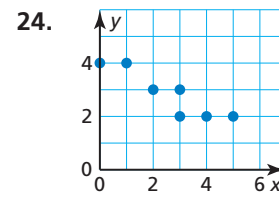
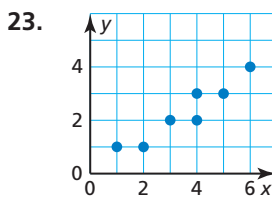
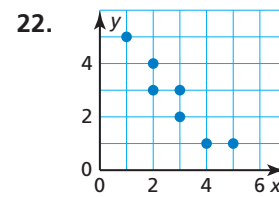
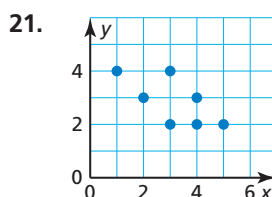
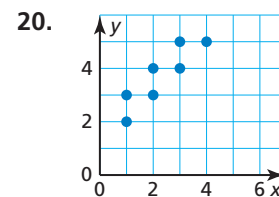
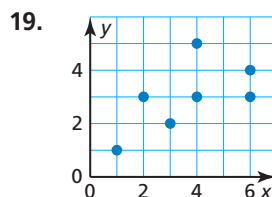
17. **MODELING WITH MATHEMATICS** The data pairs  $(x, y)$  represent the average annual tuition  $y$  (in dollars) for public colleges in the United States  $x$  years after 2005. Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit. Estimate the average annual tuition in 2020. Interpret the slope and  $y$ -intercept in this situation. (See Example 4.)

- $(0, 11,386), (1, 11,731), (2, 11,848)$   
 $(3, 12,375), (4, 12,804), (5, 13,297)$

18. **MODELING WITH MATHEMATICS** The table shows the numbers of tickets sold for a concert when different prices are charged. Write an equation of a line of fit for the data. Does it seem reasonable to use your model to predict the number of tickets sold when the ticket price is \$85? Explain.

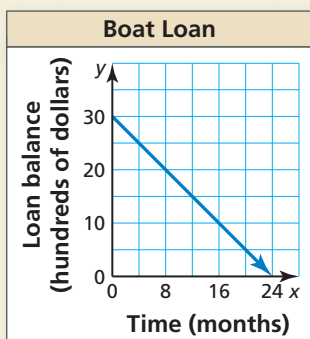
Ticket price (dollars), $x$	17	20	22	26
Tickets sold, $y$	450	423	400	395

**USING TOOLS** In Exercises 19–24, use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data. Find and interpret the correlation coefficient.



25. **OPEN-ENDED** Give two real-life quantities that have (a) a positive correlation, (b) a negative correlation, and (c) approximately no correlation. Explain.

26. **HOW DO YOU SEE IT?** You secure an interest-free loan to purchase a boat. You agree to make equal monthly payments for the next two years. The graph shows the amount of money you still owe.



- What is the slope of the line? What does the slope represent?
- What is the domain and range of the function? What does each represent?
- How much do you still owe after making payments for 12 months?

27. **MAKING AN ARGUMENT** A set of data pairs has a correlation coefficient  $r = 0.3$ . Your friend says that because the correlation coefficient is positive, it is logical to use the line of best fit to make predictions. Is your friend correct? Explain your reasoning.

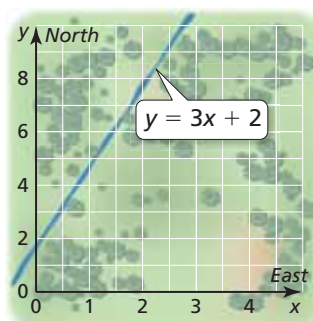
28. **THOUGHT PROVOKING** Points  $A$  and  $B$  lie on the line  $y = -x + 4$ . Choose coordinates for points  $A$ ,  $B$ , and  $C$  where point  $C$  is the same distance from point  $A$  as it is from point  $B$ . Write equations for the lines connecting points  $A$  and  $C$  and points  $B$  and  $C$ .

29. **ABSTRACT REASONING** If  $x$  and  $y$  have a positive correlation, and  $y$  and  $z$  have a negative correlation, then what can you conclude about the correlation between  $x$  and  $z$ ? Explain.

30. **MATHEMATICAL CONNECTIONS** Which equation has a graph that is a line passing through the point  $(8, -5)$  and is perpendicular to the graph of  $y = -4x + 1$ ?

- (A)  $y = \frac{1}{4}x - 5$       (B)  $y = -4x + 27$   
 (C)  $y = -\frac{1}{4}x - 7$       (D)  $y = \frac{1}{4}x - 7$

31. **PROBLEM SOLVING** You are participating in an orienteering competition. The diagram shows the position of a river that cuts through the woods. You are currently 2 miles east and 1 mile north of your starting point, the origin. What is the shortest distance you must travel to reach the river?



32. **ANALYZING RELATIONSHIPS** Data from North American countries show a positive correlation between the number of personal computers per capita and the average life expectancy in the country.

- Does a positive correlation make sense in this situation? Explain.
- Is it reasonable to conclude that giving residents of a country personal computers will lengthen their lives? Explain.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations in two variables by elimination or substitution.

(Skills Review Handbook)

33.  $3x + y = 7$   
 $-2x - y = 9$

34.  $4x + 3y = 2$   
 $2x - 3y = 1$

35.  $2x + 2y = 3$   
 $x = 4y - 1$

36.  $y = 1 + x$   
 $2x + y = -2$

37.  $\frac{1}{2}x + 4y = 4$   
 $2x - y = 1$

38.  $y = x - 4$   
 $4x + y = 26$

# 1.4 Solving Linear Systems

**Essential Question** How can you determine the number of solutions of a linear system?

A linear system is *consistent* when it has at least one solution. A linear system is *inconsistent* when it has no solution.

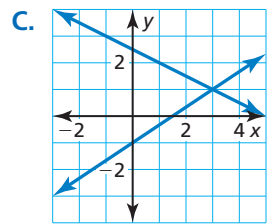
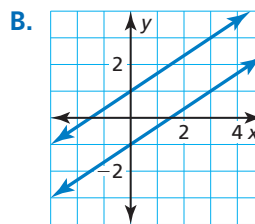
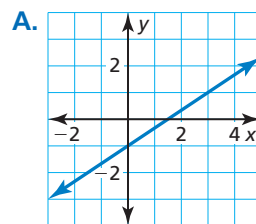
## EXPLORATION 1 Recognizing Graphs of Linear Systems

**Work with a partner.** Match each linear system with its corresponding graph. Explain your reasoning. Then classify the system as *consistent* or *inconsistent*.

a.  $2x - 3y = 3$   
 $-4x + 6y = 6$

b.  $2x - 3y = 3$   
 $x + 2y = 5$

c.  $2x - 3y = 3$   
 $-4x + 6y = -6$



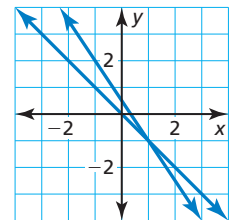
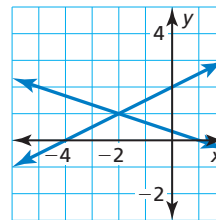
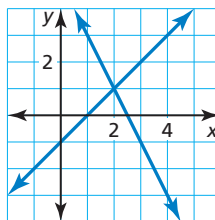
## EXPLORATION 2 Solving Systems of Linear Equations

**Work with a partner.** Solve each linear system by substitution or elimination. Then use the graph of the system below to check your solution.

a.  $2x + y = 5$   
 $x - y = 1$

b.  $x + 3y = 1$   
 $-x + 2y = 4$

c.  $x + y = 0$   
 $3x + 2y = 1$



### FINDING AN ENTRY POINT

To be proficient in math, you need to look for entry points to the solution of a problem.

### Communicate Your Answer

- How can you determine the number of solutions of a linear system?
- Suppose you were given a system of *three* linear equations in *three* variables. Explain how you would approach solving such a system.
- Apply your strategy in Question 4 to solve the linear system.

$x + y + z = 1$  Equation 1

$x - y - z = 3$  Equation 2

$-x - y + z = -1$  Equation 3

# 1.4 Lesson

## Core Vocabulary

linear equation in three variables, p. 30  
system of three linear equations, p. 30  
solution of a system of three linear equations, p. 30  
ordered triple, p. 30

### Previous

system of two linear equations

## What You Will Learn

- ▶ Visualize solutions of systems of linear equations in three variables.
- ▶ Solve systems of linear equations in three variables algebraically.
- ▶ Solve real-life problems.

## Visualizing Solutions of Systems

A **linear equation in three variables**  $x$ ,  $y$ , and  $z$  is an equation of the form  $ax + by + cz = d$ , where  $a$ ,  $b$ , and  $c$  are not all zero.

The following is an example of a **system of three linear equations** in three variables.

$$3x + 4y - 8z = -3 \quad \text{Equation 1}$$

$$x + y + 5z = -12 \quad \text{Equation 2}$$

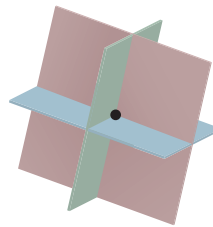
$$4x - 2y + z = 10 \quad \text{Equation 3}$$

A **solution** of such a system is an **ordered triple**  $(x, y, z)$  whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space. The graphs of three such equations that form a system are three planes whose intersection determines the number of solutions of the system, as shown in the diagrams below.

### Exactly One Solution

The planes intersect in a single point, which is the solution of the system.



### Infinitely Many Solutions

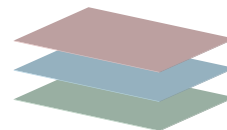
The planes intersect in a line. Every point on the line is a solution of the system.

The planes could also be the same plane. Every point in the plane is a solution of the system.



### No Solution

There are no points in common with all three planes.



## Solving Systems of Equations Algebraically

The algebraic methods you used to solve systems of linear equations in two variables can be extended to solve a system of linear equations in three variables.

### Core Concept

#### Solving a Three-Variable System

**Step 1** Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

**Step 2** Solve the new linear system for both of its variables.

**Step 3** Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as  $0 = 1$ , in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as  $0 = 0$ , the system has infinitely many solutions.

#### LOOKING FOR STRUCTURE

The coefficient of  $-1$  in Equation 3 makes  $y$  a convenient variable to eliminate.

#### EXAMPLE 1 Solving a Three-Variable System (One Solution)

Solve the system.

$$\begin{array}{rcl} 4x + 2y + 3z = 12 & \text{Equation 1} \\ 2x - 3y + 5z = -7 & \text{Equation 2} \\ 6x - y + 4z = -3 & \text{Equation 3} \end{array}$$

#### SOLUTION

**Step 1** Rewrite the system as a linear system in *two* variables.

$$\begin{array}{rcl} 4x + 2y + 3z = 12 & \text{Add 2 times Equation 3 to} \\ 12x - 2y + 8z = -6 & \text{Equation 1 (to eliminate } y\text{).} \\ \hline 16x & + 11z = 6 & \text{New Equation 1} \\ 2x - 3y + 5z = -7 & \text{Add } -3 \text{ times Equation 3 to} \\ -18x + 3y - 12z = 9 & \text{Equation 2 (to eliminate } y\text{).} \\ \hline -16x & - 7z = 2 & \text{New Equation 2} \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{rcl} 16x + 11z = 6 & \text{Add new Equation 1} \\ -16x - 7z = 2 & \text{and new Equation 2.} \\ \hline 4z = 8 & \\ z = 2 & \text{Solve for } z. \\ x = -1 & \text{Substitute into new Equation 1 or 2 to find } x. \end{array}$$

**Step 3** Substitute  $x = -1$  and  $z = 2$  into an original equation and solve for  $y$ .

$$\begin{array}{rcl} 6x - y + 4z = -3 & \text{Write original Equation 3.} \\ 6(-1) - y + 4(2) = -3 & \text{Substitute } -1 \text{ for } x \text{ and } 2 \text{ for } z. \\ y = 5 & \text{Solve for } y. \end{array}$$

► The solution is  $x = -1$ ,  $y = 5$ , and  $z = 2$ , or the ordered triple  $(-1, 5, 2)$ . Check this solution in each of the original equations.

#### ANOTHER WAY

In Step 1, you could also eliminate  $x$  to get two equations in  $y$  and  $z$ , or you could eliminate  $z$  to get two equations in  $x$  and  $y$ .



**EXAMPLE 2 Solving a Three-Variable System (No Solution)**

Solve the system.

$$\begin{array}{rcl} x + y + z = 2 & \text{Equation 1} \\ 5x + 5y + 5z = 3 & \text{Equation 2} \\ 4x + y - 3z = -6 & \text{Equation 3} \end{array}$$

**SOLUTION**

**Step 1** Rewrite the system as a linear system in *two* variables.

$$\begin{array}{rcl} -5x - 5y - 5z = -10 & \text{Add } -5 \text{ times Equation 1} \\ 5x + 5y + 5z = 3 & \text{to Equation 2.} \\ \hline 0 = -7 \end{array}$$

▶ Because you obtain a false equation, the original system has no solution.

**ANOTHER WAY**

Subtracting Equation 2 from Equation 1 gives  $z = 0$ . After substituting 0 for  $z$  in each equation, you can see that each is equivalent to  $y = x + 3$ .

**EXAMPLE 3 Solving a Three-Variable System (Many Solutions)**

Solve the system.

$$\begin{array}{rcl} x - y + z = -3 & \text{Equation 1} \\ x - y - z = -3 & \text{Equation 2} \\ 5x - 5y + z = -15 & \text{Equation 3} \end{array}$$

**SOLUTION**

**Step 1** Rewrite the system as a linear system in *two* variables.

$$\begin{array}{rcl} x - y + z = -3 & \text{Add Equation 1 to} \\ x - y - z = -3 & \text{Equation 2 (to eliminate } z\text{).} \\ \hline 2x - 2y = -6 & \text{New Equation 2} \\ x - y - z = -3 & \text{Add Equation 2 to} \\ 5x - 5y + z = -15 & \text{Equation 3 (to eliminate } z\text{).} \\ \hline 6x - 6y = -18 & \text{New Equation 3} \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{rcl} -6x + 6y = 18 & \text{Add } -3 \text{ times new Equation 2} \\ 6x - 6y = -18 & \text{to new Equation 3.} \\ \hline 0 = 0 \end{array}$$

Because you obtain the identity  $0 = 0$ , the system has infinitely many solutions.

**Step 3** Describe the solutions of the system using an ordered triple. One way to do this is to solve new Equation 2 for  $y$  to obtain  $y = x + 3$ . Then substitute  $x + 3$  for  $y$  in original Equation 1 to obtain  $z = 0$ .

▶ So, any ordered triple of the form  $(x, x + 3, 0)$  is a solution of the system.

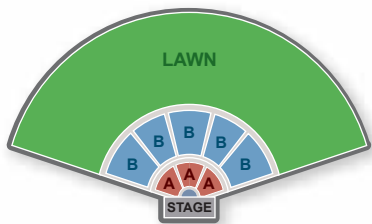
**Monitoring Progress**  [Help in English and Spanish at BigIdeasMath.com](http://BigIdeasMath.com)

Solve the system. Check your solution, if possible.

$$\begin{array}{lll} 1. \ x - 2y + z = -11 & 2. \ x + y - z = -1 & 3. \ x + y + z = 8 \\ 3x + 2y - z = 7 & 4x + 4y - 4z = -2 & x - y + z = 8 \\ -x + 2y + 4z = -9 & 3x + 2y + z = 0 & 2x + y + 2z = 16 \end{array}$$

4. In Example 3, describe the solutions of the system using an ordered triple in terms of  $y$ .

## Solving Real-Life Problems



### EXAMPLE 4 Solving a Multi-Step Problem

An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?

#### SOLUTION

**Step 1** Write a verbal model for the situation.

$$\text{Number of seats in B, } y = 3 \cdot \text{Number of seats in A, } x$$

$$\text{Number of seats in A, } x + \text{Number of seats in B, } y + \text{Number of lawn seats, } z = \text{Total number of seats}$$

$$75 \cdot \text{Number of seats in A, } x + 55 \cdot \text{Number of seats in B, } y + 30 \cdot \text{Number of lawn seats, } z = \text{Total revenue}$$

**Step 2** Write a system of equations.

$$y = 3x \quad \text{Equation 1}$$

$$x + y + z = 23,000 \quad \text{Equation 2}$$

$$75x + 55y + 30z = 870,000 \quad \text{Equation 3}$$

**Step 3** Rewrite the system in Step 2 as a linear system in *two* variables by substituting  $3x$  for  $y$  in Equations 2 and 3.

$$x + y + z = 23,000 \quad \text{Write Equation 2.}$$

$$x + 3x + z = 23,000 \quad \text{Substitute } 3x \text{ for } y.$$

$$4x + z = 23,000 \quad \text{New Equation 2}$$

$$75x + 55y + 30z = 870,000 \quad \text{Write Equation 3.}$$

$$75x + 55(3x) + 30z = 870,000 \quad \text{Substitute } 3x \text{ for } y.$$

$$240x + 30z = 870,000 \quad \text{New Equation 3}$$

**Step 4** Solve the new linear system for both of its variables.

$$\begin{array}{r} -120x - 30z = -690,000 \\ \underline{240x + 30z = 870,000} \\ 120x = 180,000 \end{array} \quad \begin{array}{l} \text{Add } -30 \text{ times new Equation 2} \\ \text{to new Equation 3.} \end{array}$$

$$x = 1500 \quad \text{Solve for } x.$$

$$y = 4500 \quad \text{Substitute into Equation 1 to find } y.$$

$$z = 17,000 \quad \text{Substitute into Equation 2 to find } z.$$

► The solution is  $x = 1500$ ,  $y = 4500$ , and  $z = 17,000$ , or  $(1500, 4500, 17,000)$ . So, there are 1500 seats in Section A, 4500 seats in Section B, and 17,000 lawn seats.

#### STUDY TIP

When substituting to find values of other variables, choose original or new equations that are easiest to use.



#### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

5. **WHAT IF?** On the first day, 10,000 tickets sold, generating \$356,000 in revenue. The number of seats sold in Sections A and B are the same. How many lawn seats are still available?

# 1.4 Exercises

Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

## Vocabulary and Core Concept Check

- VOCABULARY** The solution of a system of three linear equations is expressed as a(n) \_\_\_\_\_.
- WRITING** Explain how you know when a linear system in three variables has infinitely many solutions.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, solve the system using the elimination method. (See Example 1.)


- |   |  |
|---|--|
| 3. $x + y - 2z = 5$<br>$-x + 2y + z = 2$<br>$2x + 3y - z = 9$     | 4. $x + 4y - 6z = -1$<br>$2x - y + 2z = -7$<br>$-x + 2y - 4z = 5$    |
| 5. $2x + y - z = 9$<br>$-x + 6y + 2z = -17$<br>$5x + 7y + z = 4$  | 6. $3x + 2y - z = 8$<br>$-3x + 4y + 5z = -14$<br>$x - 3y + 4z = -14$ |
| 7. $2x + 2y + 5z = -1$<br>$2x - y + z = 2$<br>$2x + 4y - 3z = 14$ | 8. $3x + 2y - 3z = -2$<br>$7x - 2y + 5z = -14$<br>$2x + 4y + z = 6$  |


- |   |   |
|---|---|
| 13. $x + 3y - z = 2$<br>$x + y - z = 0$<br>$3x + 2y - 3z = -1$        | 14. $x + 2y - z = 3$<br>$-2x - y + z = -1$<br>$6x - 3y - z = -7$  |
| 15. $x + 2y + 3z = 4$<br>$-3x + 2y - z = 12$<br>$-2x - 2y - 4z = -14$ | 16. $-2x - 3y + z = -6$<br>$x + y - z = 5$<br>$7x + 8y - 6z = 31$ |

17. **MODELING WITH MATHEMATICS** Three orders are placed at a pizza shop. Two small pizzas, a liter of soda, and a salad cost \$14; one small pizza, a liter of soda, and three salads cost \$15; and three small pizzas, a liter of soda, and two salads cost \$22. How much does each item cost?

**ERROR ANALYSIS** In Exercises 9 and 10, describe and correct the error in the first step of solving the system of linear equations.

$$\begin{aligned} 4x - y + 2z &= -18 \\ -x + 2y + z &= 11 \\ 3x + 3y - 4z &= 44 \end{aligned}$$

9.  
$$\begin{aligned} 4x - y + 2z &= -18 \\ -4x + 2y + z &= 11 \\ \hline y + 3z &= -7 \end{aligned}$$

10.  
$$\begin{aligned} 12x - 3y + 6z &= -18 \\ 3x + 3y - 4z &= 44 \\ \hline 15x + 2z &= 26 \end{aligned}$$

In Exercises 11–16, solve the system using the elimination method. (See Examples 2 and 3.)

- |  |  |
|--|--|
| 11. $3x - y + 2z = 4$<br>$6x - 2y + 4z = -8$<br>$2x - y + 3z = 10$ | 12. $5x + y - z = 6$<br>$x + y + z = 2$<br>$12x + 4y = 10$ |
|--|--|



18. **MODELING WITH MATHEMATICS** Sam's Furniture Store places the following advertisement in the local newspaper. Write a system of equations for the three combinations of furniture. What is the price of each piece of furniture? Explain.

SALE

SAM'S Furniture Store

<p><b>\$1300</b> Sofa and love seat</p> <p><b>\$1400</b> Sofa and two chairs</p> <p><b>\$1600</b> Sofa, love seat, and one chair</p>	  
--	---

In Exercises 19–28, solve the system of linear equations using the substitution method. (See Example 4.)

19.  $-2x + y + 6z = 1$   
 $3x + 2y + 5z = 16$   
 $7x + 3y - 4z = 11$
20.  $x - 6y - 2z = -8$   
 $-x + 5y + 3z = 2$   
 $3x - 2y - 4z = 18$
21.  $x + y + z = 4$   
 $5x + 5y + 5z = 12$   
 $x - 4y + z = 9$
22.  $x + 2y = -1$   
 $-x + 3y + 2z = -4$   
 $-x + y - 4z = 10$
23.  $2x - 3y + z = 10$   
 $y + 2z = 13$   
 $z = 5$
24.  $x = 4$   
 $x + y = -6$   
 $4x - 3y + 2z = 26$
25.  $x + y - z = 4$   
 $3x + 2y + 4z = 17$   
 $-x + 5y + z = 8$
26.  $2x - y - z = 15$   
 $4x + 5y + 2z = 10$   
 $-x - 4y + 3z = -20$
27.  $4x + y + 5z = 5$   
 $8x + 2y + 10z = 10$   
 $x - y - 2z = -2$
28.  $x + 2y - z = 3$   
 $2x + 4y - 2z = 6$   
 $-x - 2y + z = -6$

29. **PROBLEM SOLVING** The number of left-handed people in the world is one-tenth the number of right-handed people. The percent of right-handed people is nine times the percent of left-handed people and ambidextrous people combined. What percent of people are ambidextrous?



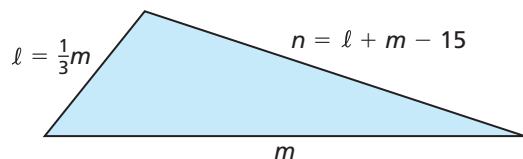
30. **MODELING WITH MATHEMATICS** Use a system of linear equations to model the data in the following newspaper article. Solve the system to find how many athletes finished in each place.

Lawrence High prevailed in Saturday's track meet with the help of 20 individual-event placers earning a combined 68 points. A first-place finish earns 5 points, a second-place finish earns 3 points, and a third-place finish earns 1 point. Lawrence had a strong second-place showing, with as many second place finishers as first- and third-place finishers combined.

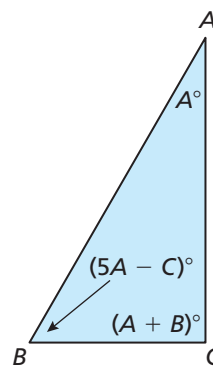
31. **WRITING** Explain when it might be more convenient to use the elimination method than the substitution method to solve a linear system. Give an example to support your claim.
32. **REPEATED REASONING** Using what you know about solving linear systems in two and three variables, plan a strategy for how you would solve a system that has four linear equations in four variables.

**MATHEMATICAL CONNECTIONS** In Exercises 33 and 34, write and use a linear system to answer the question.

33. The triangle has a perimeter of 65 feet. What are the lengths of sides  $\ell$ ,  $m$ , and  $n$ ?



34. What are the measures of angles  $A$ ,  $B$ , and  $C$ ?



35. **OPEN-ENDED** Consider the system of linear equations below. Choose nonzero values for  $a$ ,  $b$ , and  $c$  so the system satisfies the given condition. Explain your reasoning.

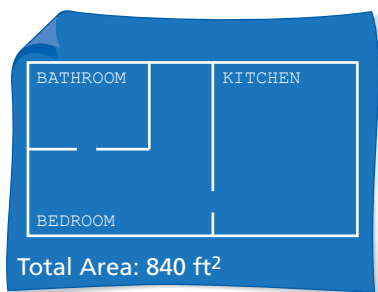
$$x + y + z = 2$$

$$ax + by + cz = 10$$

$$x - 2y + z = 4$$

- a. The system has no solution.  
 b. The system has exactly one solution.  
 c. The system has infinitely many solutions.
36. **MAKING AN ARGUMENT** A linear system in three variables has no solution. Your friend concludes that it is not possible for two of the three equations to have any points in common. Is your friend correct? Explain your reasoning.

37. **PROBLEM SOLVING** A contractor is hired to build an apartment complex. Each 840-square-foot unit has a bedroom, kitchen, and bathroom. The bedroom will be the same size as the kitchen. The owner orders 980 square feet of tile to completely cover the floors of two kitchens and two bathrooms. Determine how many square feet of carpet is needed for each bedroom.



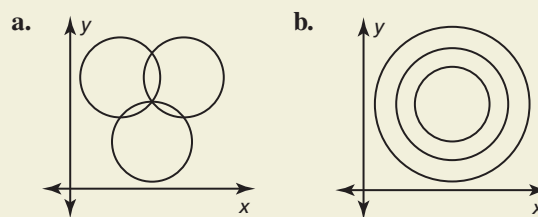
38. **THOUGHT PROVOKING** Does the system of linear equations have more than one solution? Justify your answer.

$$\begin{aligned} 4x + y + z &= 0 \\ 2x + \frac{1}{2}y - 3z &= 0 \\ -x - \frac{1}{4}y - z &= 0 \end{aligned}$$

39. **PROBLEM SOLVING** A florist must make 5 identical bridesmaid bouquets for a wedding. The budget is \$160, and each bouquet must have 12 flowers. Roses cost \$2.50 each, lilies cost \$4 each, and irises cost \$2 each. The florist wants twice as many roses as the other two types of flowers combined.

- Write a system of equations to represent this situation, assuming the florist plans to use the maximum budget.
- Solve the system to find how many of each type of flower should be in each bouquet.
- Suppose there is no limitation on the total cost of the bouquets. Does the problem still have exactly one solution? If so, find the solution. If not, give three possible solutions.

40. **HOW DO YOU SEE IT?** Determine whether the system of equations that represents the circles has *no solution*, *one solution*, or *infinitely many solutions*. Explain your reasoning.



41. **CRITICAL THINKING** Find the values of  $a$ ,  $b$ , and  $c$  so that the linear system shown has  $(-1, 2, -3)$  as its only solution. Explain your reasoning.

$$\begin{aligned} x + 2y - 3z &= a \\ -x - y + z &= b \\ 2x + 3y - 2z &= c \end{aligned}$$

42. **ANALYZING RELATIONSHIPS** Determine which arrangement(s) of the integers  $-5$ ,  $2$ , and  $3$  produce a solution of the linear system that consist of only integers. Justify your answer.

$$\begin{aligned} x - 3y + 6z &= 21 \\ \_\_x + \_\_y + \_\_z &= -30 \\ 2x - 5y + 2z &= -6 \end{aligned}$$

43. **ABSTRACT REASONING** Write a linear system to represent the first three pictures below. Use the system to determine how many tangerines are required to balance the apple in the fourth picture. *Note:* The first picture shows that one tangerine and one apple balance one grapefruit.



## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

**Simplify.** (*Skills Review Handbook*)

44.  $(x - 2)^2$       45.  $(3m + 1)^2$       46.  $(2z - 5)^2$       47.  $(4 - y)^2$

**Write a function  $g$  described by the given transformation of  $f(x) = |x| - 5$ .** (*Section 1.2*)

- translation 2 units to the left
- translation 4 units up
- reflection in the  $x$ -axis
- vertical stretch by a factor of 3

# 1.3–1.4 What Did You Learn?

## Core Vocabulary

line of fit, *p. 24*

line of best fit, *p. 25*

correlation coefficient, *p. 25*

linear equation in three variables, *p. 30*

system of three linear equations, *p. 30*

solution of a system of three linear equations, *p. 30*

ordered triple, *p. 30*

## Core Concepts

### Section 1.3

Writing an Equation of a Line, *p. 22*

Finding a Line of Fit, *p. 24*

### Section 1.4

Solving a Three-Variable System, *p. 31*

Solving Real-Life Problems, *p. 33*

## Mathematical Practices

1. Describe how you can write the equation of the line in Exercise 7 on page 26 using only one of the labeled points.
2. How did you use the information in the newspaper article in Exercise 30 on page 35 to write a system of three linear equations?
3. Explain the strategy you used to choose the values for  $a$ ,  $b$ , and  $c$  in Exercise 35 part (a) on page 35.

## Performance Task

# Secret of the Hanging Baskets

A carnival game uses two baskets hanging from springs at different heights. Next to the higher basket is a pile of baseballs. Next to the lower basket is a pile of golf balls. The object of the game is to add the same number of balls to each basket so that the baskets have the same height. But there is a catch—you only get one chance. What is the secret to winning the game?

To explore the answers to this question and more, go to [BigIdeasMath.com](http://BigIdeasMath.com).



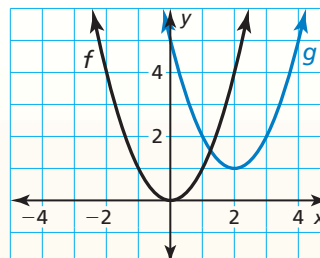


**1.1 Parent Functions and Transformations** (pp. 3–10)

Graph  $g(x) = (x - 2)^2 + 1$  and its parent function. Then describe the transformation.

The function  $g$  is a quadratic function.

- The graph of  $g$  is a translation 2 units right and 1 unit up of the graph of the parent quadratic function.



Graph the function and its parent function. Then describe the transformation.

- |                   |                      |                            |
|-------------------|----------------------|----------------------------|
| 1. $f(x) = x + 3$ | 2. $g(x) =  x  - 1$  | 3. $h(x) = \frac{1}{2}x^2$ |
| 4. $h(x) = 4$     | 5. $f(x) = - x  - 3$ | 6. $g(x) = -3(x + 3)^2$    |

**1.2 Transformations of Linear and Absolute Value Functions** (pp. 11–18)

Let the graph of  $g$  be a translation 2 units to the right followed by a reflection in the  $y$ -axis of the graph of  $f(x) = |x|$ . Write a rule for  $g$ .

**Step 1** First write a function  $h$  that represents the translation of  $f$ .

$$\begin{aligned} h(x) &= f(x - 2) && \text{Subtract 2 from the input.} \\ &= |x - 2| && \text{Replace } x \text{ with } x - 2 \text{ in } f(x). \end{aligned}$$

**Step 2** Then write a function  $g$  that represents the reflection of  $h$ .

$$\begin{aligned} g(x) &= h(-x) && \text{Multiply the input by } -1. \\ &= |-x - 2| && \text{Replace } x \text{ with } -x \text{ in } h(x). \\ &= |-(x + 2)| && \text{Factor out } -1. \\ &= |-1| \cdot |x + 2| && \text{Product Property of Absolute Value} \\ &= |x + 2| && \text{Simplify.} \end{aligned}$$

- The transformed function is  $g(x) = |x + 2|$ .

Write a function  $g$  whose graph represents the indicated transformations of the graph of  $f$ . Use a graphing calculator to check your answer.

- $f(x) = |x|$ ; reflection in the  $x$ -axis followed by a translation 4 units to the left
- $f(x) = |x|$ ; vertical shrink by a factor of  $\frac{1}{2}$  followed by a translation 2 units up
- $f(x) = x$ ; translation 3 units down followed by a reflection in the  $y$ -axis

### 1.3 Modeling with Linear Functions (pp. 21–28)

The table shows the numbers of ice cream cones sold for different outside temperatures (in degrees Fahrenheit). Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate how many ice cream cones are sold when the temperature is 60°F.

Temperature, $x$	53	62	70	82	90
Number of cones, $y$	90	105	117	131	147

**Step 1** Create a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the line that appears to most closely fit the data. One possibility is shown.

**Step 3** Choose two points on the line. For the line shown, you might choose (70, 117) and (90, 147).

**Step 4** Write an equation of the line. First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{147 - 117}{90 - 70} = \frac{30}{20} = 1.5$$

Use point-slope form to write an equation.

Use  $(x_1, y_1) = (70, 117)$ .

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 117 = 1.5(x - 70) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 117 = 1.5x - 105 \quad \text{Distributive Property}$$

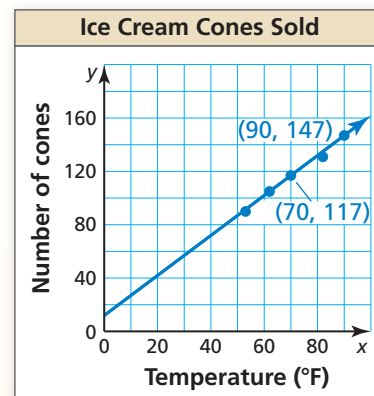
$$y = 1.5x + 12 \quad \text{Add 117 to each side.}$$

Use the equation to estimate the number of ice cream cones sold.

$$y = 1.5(60) + 12 \quad \text{Substitute 60 for } x.$$

$$= 102 \quad \text{Simplify.}$$

► Approximately 102 ice cream cones are sold when the temperature is 60°F.



**Write an equation of the line.**

**10.** The table shows the total number  $y$  (in billions) of U.S. movie admissions each year for  $x$  years. Use a graphing calculator to find an equation of the line of best fit for the data.

Year, $x$	0	2	4	6	8	10
Admissions, $y$	1.24	1.26	1.39	1.47	1.49	1.57

**11.** You ride your bike and measure how far you travel. After 10 minutes, you travel 3.5 miles. After 30 minutes, you travel 10.5 miles. Write an equation to model your distance. How far can you ride your bike in 45 minutes?

## 1.4 Solving Linear Systems (pp. 29–36)

Solve the system.

$$x - y + z = -3 \quad \text{Equation 1}$$

$$2x - y + 5z = 4 \quad \text{Equation 2}$$

$$4x + 2y - z = 2 \quad \text{Equation 3}$$

**Step 1** Rewrite the system as a linear system in two variables.

$$\begin{array}{r} x - y + z = -3 \\ 4x + 2y - z = 2 \\ \hline 5x + y = -1 \end{array} \quad \begin{array}{l} \text{Add Equation 1 to} \\ \text{Equation 3 (to eliminate } z\text{).} \end{array}$$

$$\begin{array}{r} 5x + y = -1 \\ -5x + 5y - 5z = 15 \\ \hline 2x - y + 5z = 4 \end{array} \quad \begin{array}{l} \text{New Equation 3} \\ \text{Add } -5 \text{ times Equation 1 to} \\ \text{Equation 2 (to eliminate } z\text{).} \end{array}$$

$$\begin{array}{r} 2x - y + 5z = 4 \\ -3x + 4y = 19 \end{array} \quad \begin{array}{l} \text{New Equation 2} \end{array}$$

**Step 2** Solve the new linear system for both of its variables.

$$\begin{array}{r} -20x - 4y = 4 \\ -3x + 4y = 19 \\ \hline -23x = 23 \\ x = -1 \\ y = 4 \end{array} \quad \begin{array}{l} \text{Add } -4 \text{ times new Equation 3} \\ \text{to new Equation 2.} \\ \\ \text{Solve for } x. \\ \text{Substitute into new Equation 2 or 3 to find } y. \end{array}$$

**Step 3** Substitute  $x = -1$  and  $y = 4$  into an original equation and solve for  $z$ .

$$\begin{array}{r} x - y + z = -3 \\ (-1) - 4 + z = -3 \\ z = 2 \end{array} \quad \begin{array}{l} \text{Write original Equation 1.} \\ \text{Substitute } -1 \text{ for } x \text{ and } 4 \text{ for } y. \\ \text{Solve for } z. \end{array}$$

► The solution is  $x = -1$ ,  $y = 4$ , and  $z = 2$ , or the ordered triple  $(-1, 4, 2)$ .

Solve the system. Check your solution, if possible.

12.  $x + y + z = 3$   
 $-x + 3y + 2z = -8$   
 $x = 4z$

13.  $2x - 5y - z = 17$   
 $x + y + 3z = 19$   
 $-4x + 6y + z = -20$

14.  $x + y + z = 2$   
 $2x - 3y + z = 11$   
 $-3x + 2y - 2z = -13$

15.  $x + 4y - 2z = 3$   
 $x + 3y + 7z = 1$   
 $2x + 9y - 13z = 2$

16.  $x - y + 3z = 6$   
 $x - 2y = 5$   
 $2x - 2y + 5z = 9$

17.  $x + 2z = 4$   
 $x + y + z = 6$   
 $3x + 3y + 4z = 28$

18. A school band performs a spring concert for a crowd of 600 people. The revenue for the concert is \$3150. There are 150 more adults at the concert than students. How many of each type of ticket are sold?

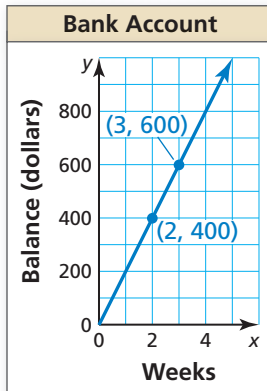


# 1

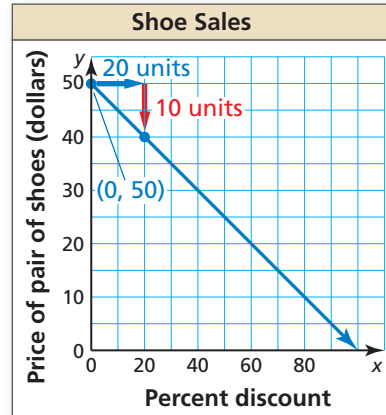
# Chapter Test

Write an equation of the line and interpret the slope and y-intercept.

1.



2.



Solve the system. Check your solution, if possible.

3.  $-2x + y + 4z = 5$

$x + 3y - z = 2$

$4x + y - 6z = 11$

4.  $y = \frac{1}{2}z$

$x + 2y + 5z = 2$

$3x + 6y - 3z = 9$

5.  $x - y + 5z = 3$

$2x + 3y - z = 2$

$-4x - y - 9z = -8$

Graph the function and its parent function. Then describe the transformation.

6.  $f(x) = |x - 1|$

7.  $f(x) = (3x)^2$

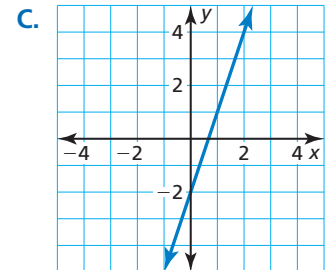
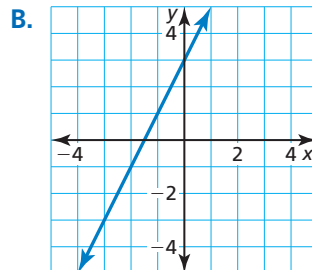
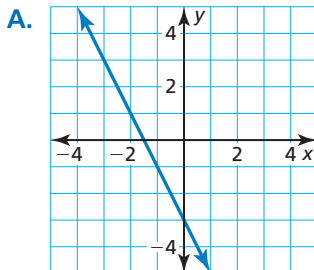
8.  $f(x) = 4$

Match the transformation of  $f(x) = x$  with its graph. Then write a rule for  $g$ .

9.  $g(x) = 2f(x) + 3$

10.  $g(x) = 3f(x) - 2$

11.  $g(x) = -2f(x) - 3$



12. A bakery sells doughnuts, muffins, and bagels. The bakery makes three times as many doughnuts as bagels. The bakery earns a total of \$150 when all 130 baked items in stock are sold. How many of each item are in stock? Justify your answer.

*Breakfast Specials*

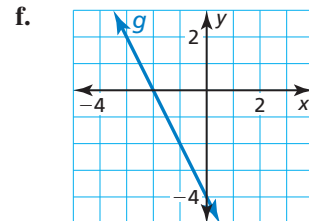
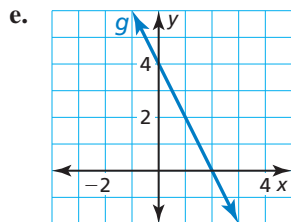
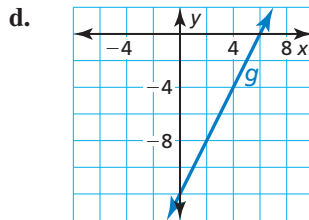
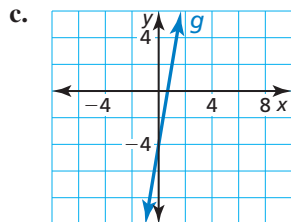
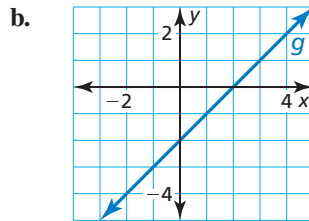
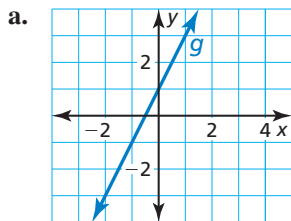
Doughnuts.....	\$1.00	
Muffins .....	\$1.50	
Bagels.....	\$1.20	

13. A fountain with a depth of 5 feet is drained and then refilled. The water level (in feet) after  $t$  minutes can be modeled by  $f(t) = \frac{1}{4}|t - 20|$ . A second fountain with the same depth is drained and filled twice as quickly as the first fountain. Describe how to transform the graph of  $f$  to model the water level in the second fountain after  $t$  minutes. Find the depth of each fountain after 4 minutes. Justify your answers.

# 1

## Cumulative Assessment

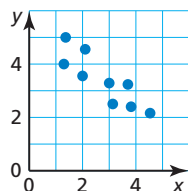
1. Describe the transformation of the graph of  $f(x) = 2x - 4$  represented in each graph.



2. The table shows the tuition costs for a private school between the years 2010 and 2013.

Years after 2010, $x$	0	1	2	3
Tuition (dollars), $y$	36,208	37,620	39,088	40,594

- Verify that the data show a linear relationship. Then write an equation of a line of fit.
  - Interpret the slope and  $y$ -intercept in this situation.
  - Predict the cost of tuition in 2015.
3. Your friend claims the line of best fit for the data shown in the scatter plot has a correlation coefficient close to 1. Is your friend correct? Explain your reasoning.



4. Order the following linear systems from least to greatest according to the number of solutions.

A.  $2x + 4y - z = 7$   
 $14x + 28y - 7z = 49$   
 $-x + 6y + 12z = 13$

B.  $3x - 3y + 3z = 5$   
 $-x + y - z = 8$   
 $14x - 3y + 12z = 108$

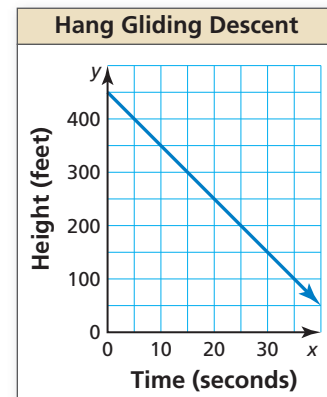
C.  $4x - y + 2z = 18$   
 $-x + 2y + z = 11$   
 $3x + 3y - 4z = 44$

5. You make DVDs of three types of shows: comedy, drama, and reality-based. An episode of a comedy lasts 30 minutes, while a drama and a reality-based episode each last 60 minutes. The DVDs can hold 360 minutes of programming.

- a. You completely fill a DVD with seven episodes and include twice as many episodes of a drama as a comedy. Create a system of equations that models the situation.
- b. How many episodes of each type of show are on the DVD in part (a)?
- c. You completely fill a second DVD with only six episodes. Do the two DVDs have a different number of comedies? dramas? reality-based episodes? Explain.

6. The graph shows the height of a hang glider over time. Which equation models the situation?

- (A)  $y + 450 = 10x$   
 (B)  $10y = -x + 450$   
 (C)  $\frac{1}{10}y = -x + 450$   
 (D)  $10x + y = 450$



7. Let  $f(x) = x$  and  $g(x) = -3x - 4$ . Select the possible transformations (in order) of the graph of  $f$  represented by the function  $g$ .

- (A) reflection in the  $x$ -axis  
 (B) reflection in the  $y$ -axis  
 (C) vertical translation 4 units down  
 (D) horizontal translation 4 units right  
 (E) horizontal shrink by a factor of  $\frac{1}{3}$   
 (F) vertical stretch by a factor of 3

8. Choose the correct equality or inequality symbol which completes the statement below about the linear functions  $f$  and  $g$ . Explain your reasoning.

$x$	$f(x)$
-5	-23
-4	-20
-3	-17
-2	-14

$x$	$g(x)$
-2	-18
-1	-14
0	-10
1	-6

$f(22)$     $g(22)$