

3 Quadratic Equations and Complex Numbers

- 3.1 Solving Quadratic Equations
- 3.2 Complex Numbers
- 3.3 Completing the Square
- 3.4 Using the Quadratic Formula
- 3.5 Solving Nonlinear Systems
- 3.6 Quadratic Inequalities



Robot-Building Competition (p. 145)



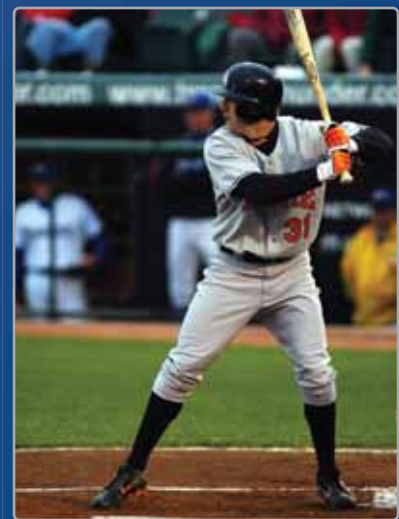
Broadcast Tower (p. 137)



Feeding Gannet (p. 129)



Electrical Circuits (p. 106)



Baseball (p. 115)

Maintaining Mathematical Proficiency

Simplifying Square Roots

Example 1 Simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \cdot 2}$$

Factor using the greatest perfect square factor.

$$= \sqrt{4} \cdot \sqrt{2}$$

Product Property of Square Roots

$$= 2\sqrt{2}$$

Simplify.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}, \text{ where } a, b \geq 0$$

Example 2 Simplify $\sqrt{\frac{7}{36}}$.

$$\sqrt{\frac{7}{36}} = \frac{\sqrt{7}}{\sqrt{36}}$$

Quotient Property of Square Roots

$$= \frac{\sqrt{7}}{6}$$

Simplify.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \text{ where } a \geq 0 \text{ and } b > 0$$

Simplify the expression.

1. $\sqrt{27}$

2. $-\sqrt{112}$

3. $\sqrt{\frac{11}{64}}$

4. $\sqrt{\frac{147}{100}}$

5. $\sqrt{\frac{18}{49}}$

6. $-\sqrt{\frac{65}{121}}$

7. $-\sqrt{80}$

8. $\sqrt{32}$

Factoring Special Products

Example 3 Factor (a) $x^2 - 4$ and (b) $x^2 - 14x + 49$.

a. $x^2 - 4 = x^2 - 2^2$

Write as $a^2 - b^2$.

$$= (x + 2)(x - 2)$$

Difference of Two Squares Pattern

► So, $x^2 - 4 = (x + 2)(x - 2)$.

b. $x^2 - 14x + 49 = x^2 - 2(x)(7) + 7^2$

Write as $a^2 - 2ab + b^2$.

$$= (x - 7)^2$$

Perfect Square Trinomial Pattern

► So, $x^2 - 14x + 49 = (x - 7)^2$.

Factor the polynomial.

9. $x^2 - 36$

10. $x^2 - 9$

11. $4x^2 - 25$

12. $x^2 - 22x + 121$

13. $x^2 + 28x + 196$

14. $49x^2 + 210x + 225$

15. **ABSTRACT REASONING** Determine the possible integer values of a and c for which the trinomial $ax^2 + 8x + c$ is factorable using the Perfect Square Trinomial Pattern. Explain your reasoning.

Recognizing the Limitations of Technology

Core Concept

Graphing Calculator Limitations

Graphing calculators have a limited number of pixels to display the graph of a function. The result may be an inaccurate or misleading graph.

To correct this issue, use a viewing window setting based on the dimensions of the screen (in pixels).

EXAMPLE 1 Recognizing an Incorrect Graph

Use a graphing calculator to draw the circle given by the equation $x^2 + y^2 = 6.25$.

SOLUTION

Begin by solving the equation for y .

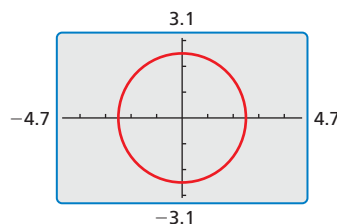
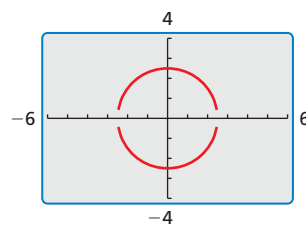
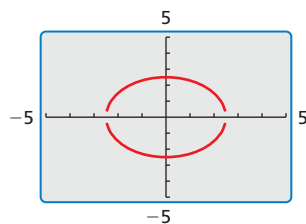
$$y = \sqrt{6.25 - x^2} \quad \text{Equation of upper semicircle}$$

$$y = -\sqrt{6.25 - x^2} \quad \text{Equation of lower semicircle}$$

The graphs of these two equations are shown in the first viewing window. Notice that there are two issues. First, the graph resembles an oval rather than a circle. Second, the two parts of the graph appear to have gaps between them.

You can correct the first issue by using a *square viewing window*, as shown in the second viewing window.

To correct the second issue, you need to know the dimensions of the graphing calculator screen in terms of the number of pixels. For instance, for a screen that is 63 pixels high and 95 pixels wide, use a viewing window setting as shown at the right.



Monitoring Progress

1. Explain why the second viewing window in Example 1 shows gaps between the upper and lower semicircles, but the third viewing window does not show gaps.

Use a graphing calculator to draw an accurate graph of the equation. Explain your choice of viewing window.

2. $y = \sqrt{x^2 - 1.5}$
3. $y = \sqrt{x - 2.5}$
4. $x^2 + y^2 = 12.25$
5. $x^2 + y^2 = 20.25$
6. $x^2 + 4y^2 = 12.25$
7. $4x^2 + y^2 = 20.25$

Mathematical Practices

Mathematically proficient students **make sound decisions** about when technological tools might be helpful, recognizing both the insight gained and the limitations. (MP5)

Recognizing the Limitations of Technology

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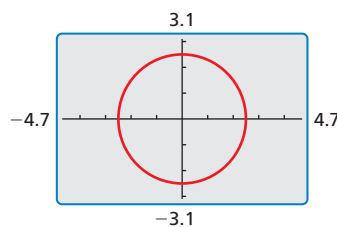
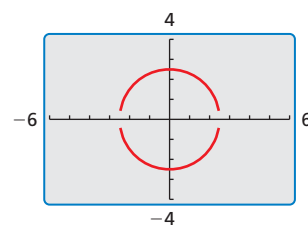
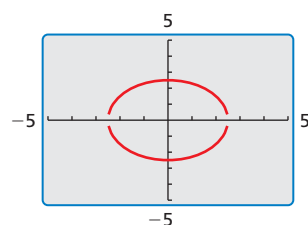
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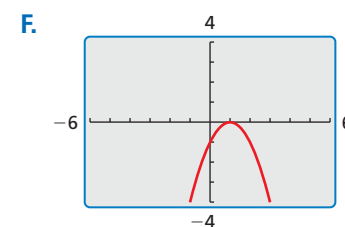
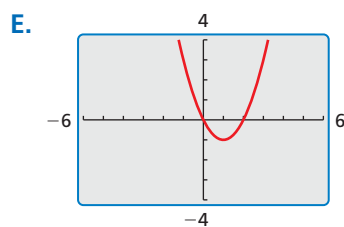
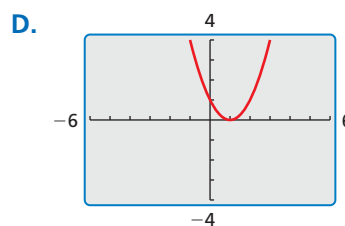
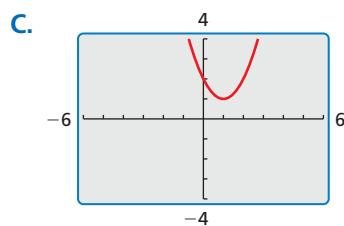
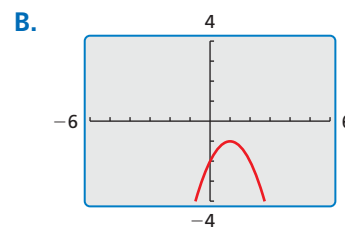
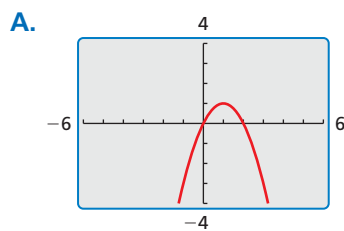
3.1 Solving Quadratic Equations

Essential Question How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

EXPLORATION 1 Matching a Quadratic Function with Its Graph

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Determine the number of x -intercepts of the graph.

- a. $f(x) = x^2 - 2x$ b. $f(x) = x^2 - 2x + 1$ c. $f(x) = x^2 - 2x + 2$
 d. $f(x) = -x^2 + 2x$ e. $f(x) = -x^2 + 2x - 1$ f. $f(x) = -x^2 + 2x - 2$



EXPLORATION 2 Solving Quadratic Equations

Work with a partner. Use the results of Exploration 1 to find the real solutions (if any) of each quadratic equation.

- a. $x^2 - 2x = 0$ b. $x^2 - 2x + 1 = 0$ c. $x^2 - 2x + 2 = 0$
 d. $-x^2 + 2x = 0$ e. $-x^2 + 2x - 1 = 0$ f. $-x^2 + 2x - 2 = 0$

Communicate Your Answer

- How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?
- How many real solutions does the quadratic equation $x^2 + 3x + 2 = 0$ have? How do you know? What are the solutions?

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to make conjectures about the form and meaning of solutions.

3.1 Lesson

Core Vocabulary

quadratic equation in one variable, p. 94
 root of an equation, p. 94
 zero of a function, p. 96

Previous

properties of square roots
 factoring
 rationalizing the denominator

STUDY TIP

Quadratic equations can have zero, one, or two real solutions.

What You Will Learn

- ▶ Solve quadratic equations by graphing.
- ▶ Solve quadratic equations algebraically.
- ▶ Solve real-life problems.

Solving Quadratic Equations by Graphing

A **quadratic equation in one variable** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$.

A **root of an equation** is a solution of the equation. You can use various methods to solve quadratic equations.

Core Concept

Solving Quadratic Equations

By graphing

Find the x -intercepts of the related function $y = ax^2 + bx + c$.

Using square roots

Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring

Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

EXAMPLE 1 Solving Quadratic Equations by Graphing

Solve each equation by graphing.

a. $x^2 - x - 6 = 0$

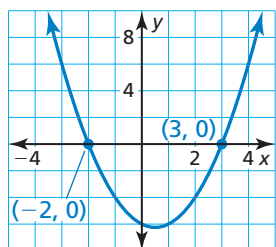
b. $-2x^2 - 2 = 4x$

SOLUTION

a. The equation is in standard form.

Graph the related function

$$y = x^2 - x - 6.$$



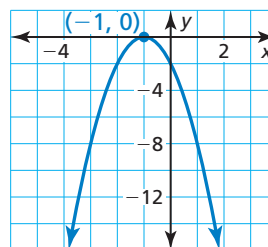
The x -intercepts are -2 and 3 .

- ▶ The solutions, or roots, are $x = -2$ and $x = 3$.

b. Add $-4x$ to each side to obtain

$$-2x^2 - 4x - 2 = 0.$$

Graph the related function $y = -2x^2 - 4x - 2$.



The x -intercept is -1 .

- ▶ The solution, or root, is $x = -1$.

Check

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (-2)^2 - (-2) - 6 &\stackrel{?}{=} 0 \\ 4 + 2 - 6 &\stackrel{?}{=} 0 \\ 0 &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} x^2 - x - 6 &= 0 \\ 3^2 - 3 - 6 &\stackrel{?}{=} 0 \\ 9 - 3 - 6 &\stackrel{?}{=} 0 \\ 0 &= 0 \quad \checkmark \end{aligned}$$

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Solve the equation by graphing.

1. $x^2 - 8x + 12 = 0$
2. $4x^2 - 12x + 9 = 0$
3. $\frac{1}{2}x^2 = 6x - 20$

Solving Quadratic Equations Algebraically

When solving quadratic equations using square roots, you can use properties of square roots to write your solutions in different forms.

When a radicand in the denominator of a fraction is not a perfect square, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called *rationalizing the denominator*.

EXAMPLE 2 Solving Quadratic Equations Using Square Roots

Solve each equation using square roots.

a. $4x^2 - 31 = 49$

b. $3x^2 + 9 = 0$

c. $\frac{2}{5}(x + 3)^2 = 5$

SOLUTION

a. $4x^2 - 31 = 49$

$$4x^2 = 80$$

$$x^2 = 20$$

$$x = \pm\sqrt{20}$$

$$x = \pm\sqrt{4} \cdot \sqrt{5}$$

$$x = \pm 2\sqrt{5}$$

Write the equation.

Add 31 to each side.

Divide each side by 4.

Take square root of each side.

Product Property of Square Roots

Simplify.

▶ The solutions are $x = 2\sqrt{5}$ and $x = -2\sqrt{5}$.

b. $3x^2 + 9 = 0$

$$3x^2 = -9$$

$$x^2 = -3$$

Write the equation.

Subtract 9 from each side.

Divide each side by 3.

▶ The square of a real number cannot be negative. So, the equation has no real solution.

c. $\frac{2}{5}(x + 3)^2 = 5$

$$(x + 3)^2 = \frac{25}{2}$$

$$x + 3 = \pm\sqrt{\frac{25}{2}}$$

$$x = -3 \pm \sqrt{\frac{25}{2}}$$

$$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}}$$

$$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = -3 \pm \frac{5\sqrt{2}}{2}$$

Write the equation.

Multiply each side by $\frac{5}{2}$.

Take square root of each side.

Subtract 3 from each side.

Quotient Property of Square Roots

Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

Simplify.

▶ The solutions are $x = -3 + \frac{5\sqrt{2}}{2}$ and $x = -3 - \frac{5\sqrt{2}}{2}$.

LOOKING FOR STRUCTURE

Notice that $(x + 3)^2 = \frac{25}{2}$ is of the form $u^2 = d$, where $u = x + 3$.

STUDY TIP

Because $\frac{\sqrt{2}}{\sqrt{2}} = 1$, the value of $\frac{\sqrt{25}}{\sqrt{2}}$ does not change when you multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

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Solve the equation using square roots.

4. $\frac{2}{3}x^2 + 14 = 20$

5. $-2x^2 + 1 = -6$

6. $2(x - 4)^2 = -5$

When the left side of $ax^2 + bx + c = 0$ is factorable, you can solve the equation using the *Zero-Product Property*.

Core Concept

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

EXAMPLE 3 Solving a Quadratic Equation by Factoring

Solve $x^2 - 4x = 45$ by factoring.

SOLUTION

$$\begin{aligned} x^2 - 4x &= 45 \\ x^2 - 4x - 45 &= 0 \\ (x - 9)(x + 5) &= 0 \\ x - 9 &= 0 \quad \text{or} \quad x + 5 = 0 \\ x &= 9 \quad \text{or} \quad x = -5 \end{aligned}$$

Write the equation.
Write in standard form.
Factor the polynomial.
Zero-Product Property
Solve for x .

▶ The solutions are $x = -5$ and $x = 9$.

You know the x -intercepts of the graph of $f(x) = a(x - p)(x - q)$ are p and q . Because the value of the function is zero when $x = p$ and when $x = q$, the numbers p and q are also called *zeros* of the function. A **zero of a function** f is an x -value for which $f(x) = 0$.

EXAMPLE 4 Finding the Zeros of a Quadratic Function

Find the zeros of $f(x) = 2x^2 - 11x + 12$.

SOLUTION

To find the zeros of the function, find the x -values for which $f(x) = 0$.

$$\begin{aligned} 2x^2 - 11x + 12 &= 0 \\ (2x - 3)(x - 4) &= 0 \\ 2x - 3 &= 0 \quad \text{or} \quad x - 4 = 0 \\ x &= 1.5 \quad \text{or} \quad x = 4 \end{aligned}$$

Set $f(x)$ equal to 0.
Factor the polynomial.
Zero-Product Property
Solve for x .

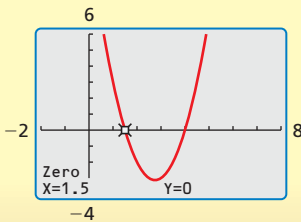
▶ The zeros of the function are $x = 1.5$ and $x = 4$. You can check this by graphing the function. The x -intercepts are 1.5 and 4.

UNDERSTANDING MATHEMATICAL TERMS

If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x -intercept of the graph of the function, and k is also a root of the equation $ax^2 + bx + c = 0$.



Check



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Solve the equation by factoring.

7. $x^2 + 12x + 35 = 0$

8. $3x^2 - 5x = 2$

Find the zero(s) of the function.

9. $f(x) = x^2 - 8x$

10. $f(x) = 4x^2 + 28x + 49$

Solving Real-Life Problems

To find the maximum value or minimum value of a quadratic function, you can first use factoring to write the function in intercept form $f(x) = a(x - p)(x - q)$. Because the vertex of the function lies on the axis of symmetry, $x = \frac{p + q}{2}$, the maximum value or minimum value occurs at the average of the zeros p and q .

EXAMPLE 5 Solving a Multi-Step Problem

A monthly teen magazine has 48,000 subscribers when it charges \$20 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?



SOLUTION

Step 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

Step 2 Write a verbal model. Then write and simplify a quadratic function.



Annual revenue (dollars)	=	Number of subscribers (people)	•	Subscription price (dollars/person)
↓		↓		↓
$R(x)$	=	$(48,000 - 2000x)$	•	$(20 + x)$
$R(x)$	=	$(-2000x + 48,000)(x + 20)$		
$R(x)$	=	$-2000(x - 24)(x + 20)$		

Step 3 Identify the zeros and find their average. Then find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 24 and -20 . The average of the zeros is $\frac{24 + (-20)}{2} = 2$.

To maximize revenue, each subscription should cost $\$20 + \$2 = \$22$.

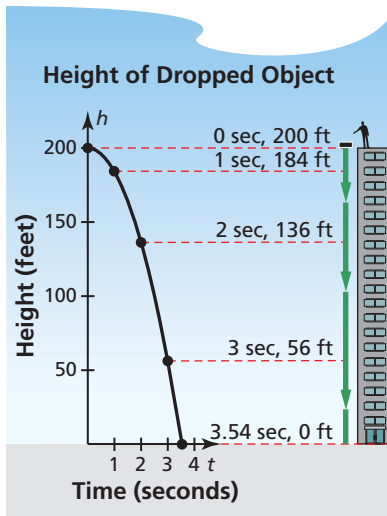
Step 4 Find the maximum annual revenue.

$$R(2) = -2000(2 - 24)(2 + 20) = \$968,000$$

► So, the magazine should charge \$22 per subscription to maximize annual revenue. The maximum annual revenue is \$968,000.

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11. **WHAT IF?** The magazine initially charges \$21 per annual subscription. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?



When an object is dropped, its height h (in feet) above the ground after t seconds can be modeled by the function $h = -16t^2 + h_0$, where h_0 is the initial height (in feet) of the object. The graph of $h = -16t^2 + 200$, representing the height of an object dropped from an initial height of 200 feet, is shown at the left.

The model $h = -16t^2 + h_0$ assumes that the force of air resistance on the object is negligible. Also, this model applies only to objects dropped on Earth. For planets with stronger or weaker gravitational forces, different models are used.

EXAMPLE 6 Modeling a Dropped Object

For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet.

- Write a function that gives the height h (in feet) of the container after t seconds. How long does the container take to hit the ground?
- Find and interpret $h(1) - h(1.5)$.

SOLUTION

- The initial height is 50, so the model is $h = -16t^2 + 50$. Find the zeros of the function.

$$h = -16t^2 + 50$$

Write the function.

$$0 = -16t^2 + 50$$

Substitute 0 for h .

$$-50 = -16t^2$$

Subtract 50 from each side.

$$\frac{-50}{-16} = t^2$$

Divide each side by -16 .

$$\pm \sqrt{\frac{50}{16}} = t$$

Take square root of each side.

$$\pm 1.8 \approx t$$

Use a calculator.

- Reject the negative solution, -1.8 , because time must be positive. The container will fall for about 1.8 seconds before it hits the ground.

- Find $h(1)$ and $h(1.5)$. These represent the heights after 1 and 1.5 seconds.

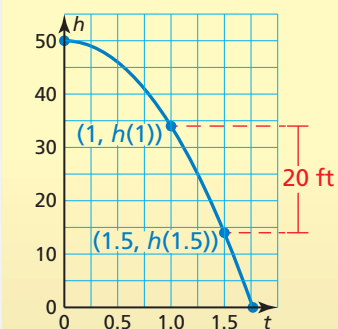
$$h(1) = -16(1)^2 + 50 = -16 + 50 = 34$$

$$h(1.5) = -16(1.5)^2 + 50 = -16(2.25) + 50 = -36 + 50 = 14$$

$$h(1) - h(1.5) = 34 - 14 = 20$$

- So, the container fell 20 feet between 1 and 1.5 seconds. You can check this by graphing the function. The points appear to be about 20 feet apart. So, the answer is reasonable.

Check



INTERPRETING EXPRESSIONS

In the model for the height of a dropped object, the term $-16t^2$ indicates that an object has fallen $16t^2$ feet after t seconds.

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12. **WHAT IF?** The egg container is dropped from a height of 80 feet. How does this change your answers in parts (a) and (b)?

3.1 Exercises

Vocabulary and Core Concept Check

- WRITING** Explain how to use graphing to find the roots of the equation $ax^2 + bx + c = 0$.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What are the zeros of $f(x) = x^2 + 3x - 10$?

What are the solutions of $x^2 + 3x - 10 = 0$?

What are the roots of $10 - x^2 = 3x$?

What is the y-intercept of the graph of $y = (x + 5)(x - 2)$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the equation by graphing.
(See Example 1.)

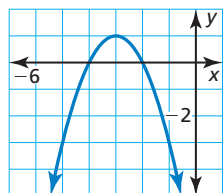
- | | |
|-------------------------------|-------------------------------|
| 3. $x^2 + 3x + 2 = 0$ | 4. $-x^2 + 2x + 3 = 0$ |
| 5. $0 = x^2 - 9$ | 6. $-8 = -x^2 - 4$ |
| 7. $8x = -4 - 4x^2$ | 8. $3x^2 = 6x - 3$ |
| 9. $7 = -x^2 - 4x$ | 10. $2x = x^2 + 2$ |
| 11. $\frac{1}{5}x^2 + 6 = 2x$ | 12. $3x = \frac{1}{4}x^2 + 5$ |

In Exercises 13–20, solve the equation using square roots. (See Example 2.)

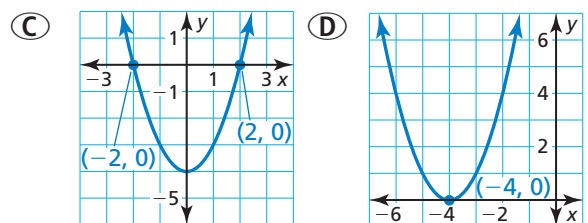
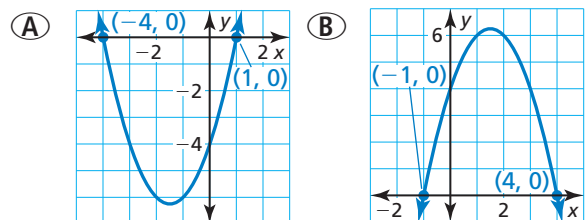
- | | |
|--|---|
| 13. $s^2 = 144$ | 14. $a^2 = 81$ |
| 15. $(z - 6)^2 = 25$ | 16. $(p - 4)^2 = 49$ |
| 17. $4(x - 1)^2 + 2 = 10$ | 18. $2(x + 2)^2 - 5 = 8$ |
| 19. $\frac{1}{2}r^2 - 10 = \frac{3}{2}r^2$ | 20. $\frac{1}{5}x^2 + 2 = \frac{3}{5}x^2$ |

21. **ANALYZING RELATIONSHIPS** Which equations have roots that are equivalent to the x-intercepts of the graph shown?

- (A) $-x^2 - 6x - 8 = 0$
 (B) $0 = (x + 2)(x + 4)$
 (C) $0 = -(x + 2)^2 + 4$
 (D) $2x^2 - 4x - 6 = 0$
 (E) $4(x + 3)^2 - 4 = 0$



22. **ANALYZING RELATIONSHIPS** Which graph has x-intercepts that are equivalent to the roots of the equation $(x - \frac{3}{2})^2 = \frac{25}{4}$? Explain your reasoning.



ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in solving the equation.

23.
$$\begin{aligned} 2(x + 1)^2 + 3 &= 21 \\ 2(x + 1)^2 &= 18 \\ (x + 1)^2 &= 9 \\ x + 1 &= 3 \\ x &= 2 \end{aligned}$$

24.
$$\begin{aligned} -2x^2 - 8 &= 0 \\ -2x^2 &= 8 \\ x^2 &= -4 \\ x &= \pm 2 \end{aligned}$$

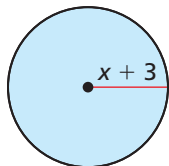
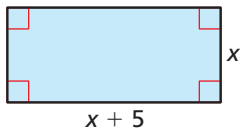
25. **OPEN-ENDED** Write an equation of the form $x^2 = d$ that has (a) two real solutions, (b) one real solution, and (c) no real solution.
26. **ANALYZING EQUATIONS** Which equation has one real solution? Explain.
- (A) $3x^2 + 4 = -2(x^2 + 8)$
- (B) $5x^2 - 4 = x^2 - 4$
- (C) $2(x + 3)^2 = 18$
- (D) $\frac{3}{2}x^2 - 5 = 19$

In Exercises 27–34, solve the equation by factoring. (See Example 3.)

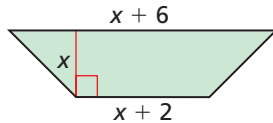
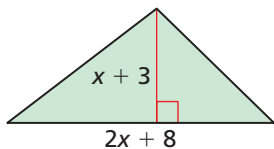
27. $0 = x^2 + 6x + 9$ 28. $0 = z^2 - 10z + 25$
29. $x^2 - 8x = -12$ 30. $x^2 - 11x = -30$
31. $n^2 - 6n = 0$ 32. $a^2 - 49 = 0$
33. $2w^2 - 16w = 12w - 48$
34. $-y + 28 + y^2 = 2y + 2y^2$

MATHEMATICAL CONNECTIONS In Exercises 35–38, find the value of x .

35. Area of rectangle = 36 36. Area of circle = 25π



37. Area of triangle = 42 38. Area of trapezoid = 32



In Exercises 39–46, solve the equation using any method. Explain your reasoning.

39. $u^2 = -9u$ 40. $\frac{t^2}{20} + 8 = 15$
41. $-(x + 9)^2 = 64$ 42. $-2(x + 2)^2 = 5$
43. $7(x - 4)^2 - 18 = 10$ 44. $t^2 + 8t + 16 = 0$
45. $x^2 + 3x + \frac{5}{4} = 0$ 46. $x^2 - 1.75 = 0.5$

In Exercises 47–54, find the zero(s) of the function. (See Example 4.)

47. $g(x) = x^2 + 6x + 8$ 48. $f(x) = x^2 - 8x + 16$
49. $h(x) = x^2 + 7x - 30$ 50. $g(x) = x^2 + 11x$
51. $f(x) = 2x^2 - 2x - 12$ 52. $f(x) = 4x^2 - 12x + 9$
53. $g(x) = x^2 + 22x + 121$
54. $h(x) = x^2 + 19x + 84$
55. **REASONING** Write a quadratic function in the form $f(x) = x^2 + bx + c$ that has zeros 8 and 11.

56. **NUMBER SENSE** Write a quadratic equation in standard form that has roots equidistant from 10 on the number line.
57. **PROBLEM SOLVING** A restaurant sells 330 sandwiches each day. For each \$0.25 decrease in price, the restaurant sells about 15 more sandwiches. How much should the restaurant charge to maximize daily revenue? What is the maximum daily revenue? (See Example 5.)

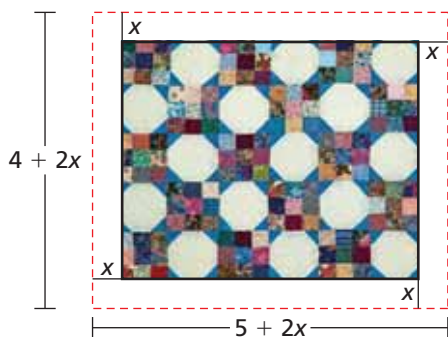


58. **PROBLEM SOLVING** An athletic store sells about 200 pairs of basketball shoes per month when it charges \$120 per pair. For each \$2 increase in price, the store sells two fewer pairs of shoes. How much should the store charge to maximize monthly revenue? What is the maximum monthly revenue?
59. **MODELING WITH MATHEMATICS** Niagara Falls is made up of three waterfalls. The height of the Canadian Horseshoe Falls is about 188 feet above the lower Niagara River. A log falls from the top of Horseshoe Falls. (See Example 6.)
- a. Write a function that gives the height h (in feet) of the log after t seconds. How long does the log take to reach the river?
- b. Find and interpret $h(2) - h(3)$.

60. **MODELING WITH MATHEMATICS** According to legend, in 1589, the Italian scientist Galileo Galilei dropped rocks of different weights from the top of the Leaning Tower of Pisa to prove his conjecture that the rocks would hit the ground at the same time. The height h (in feet) of a rock after t seconds can be modeled by $h(t) = 196 - 16t^2$.



- Find and interpret the zeros of the function. Then use the zeros to sketch the graph.
 - What do the domain and range of the function represent in this situation?
61. **PROBLEM SOLVING** You make a rectangular quilt that is 5 feet by 4 feet. You use the remaining 10 square feet of fabric to add a border of uniform width to the quilt. What is the width of the border?

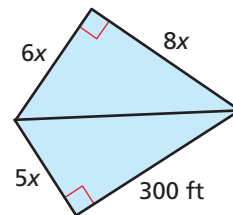


62. **MODELING WITH MATHEMATICS** You drop a seashell into the ocean from a height of 40 feet. Write an equation that models the height h (in feet) of the seashell above the water after t seconds. How long is the seashell in the air?
63. **WRITING** The equation $h = 0.019s^2$ models the height h (in feet) of the largest ocean waves when the wind speed is s knots. Compare the wind speeds required to generate 5-foot waves and 20-foot waves.



64. **CRITICAL THINKING** Write and solve an equation to find two consecutive odd integers whose product is 143.

65. **MATHEMATICAL CONNECTIONS** A quadrilateral is divided into two right triangles as shown in the figure. What is the length of each side of the quadrilateral?



66. **ABSTRACT REASONING** Suppose the equation $ax^2 + bx + c = 0$ has no real solution and a graph of the related function has a vertex that lies in the second quadrant.

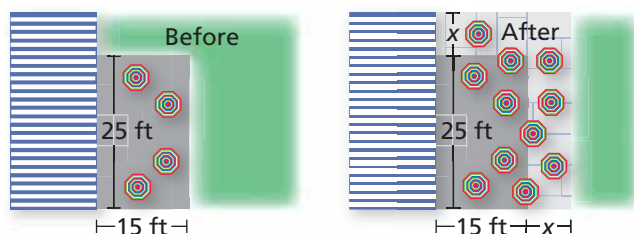
- Is the value of a positive or negative? Explain your reasoning.
- Suppose the graph is translated so the vertex is in the fourth quadrant. Does the graph have any x -intercepts? Explain.

67. **REASONING** When an object is dropped on *any* planet, its height h (in feet) after t seconds can be modeled by the function $h = -\frac{g}{2}t^2 + h_0$, where h_0 is the object's initial height and g is the planet's acceleration due to gravity. Suppose a rock is dropped from the same initial height on the three planets shown. Make a conjecture about which rock will hit the ground first. Justify your answer.



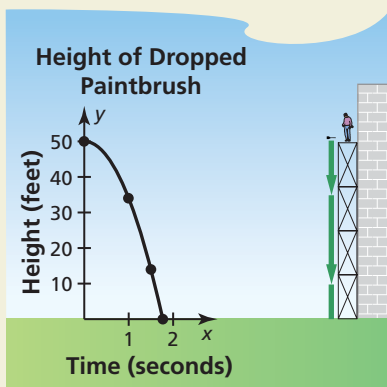
Earth: $g = 32 \text{ ft/sec}^2$ Mars: $g = 12 \text{ ft/sec}^2$ Jupiter: $g = 76 \text{ ft/sec}^2$

68. **PROBLEM SOLVING** A café has an outdoor, rectangular patio. The owner wants to add 329 square feet to the area of the patio by expanding the existing patio as shown. Write and solve an equation to find the value of x . By what distance should the patio be extended?



69. **PROBLEM SOLVING** A flea can jump very long distances. The path of the jump of a flea can be modeled by the graph of the function $y = -0.189x^2 + 2.462x$, where x is the horizontal distance (in inches) and y is the vertical distance (in inches). Graph the function. Identify the vertex and zeros and interpret their meanings in this situation.

70. **HOW DO YOU SEE IT?** An artist is painting a mural and drops a paintbrush. The graph represents the height h (in feet) of the paintbrush after t seconds.



- What is the initial height of the paintbrush?
- How long does it take the paintbrush to reach the ground? Explain.

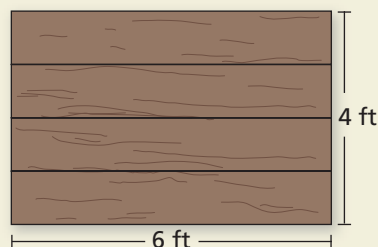
71. **MAKING AN ARGUMENT** Your friend claims the equation $x^2 + 7x = -49$ can be solved by factoring and has a solution of $x = 7$. You solve the equation by graphing the related function and claim there is no solution. Who is correct? Explain.

72. **ABSTRACT REASONING** Factor the expressions $x^2 - 4$ and $x^2 - 9$. Recall that an expression in this form is called a difference of two squares. Use your answers to factor the expression $x^2 - a^2$. Graph the related function $y = x^2 - a^2$. Label the vertex, x -intercepts, and axis of symmetry.

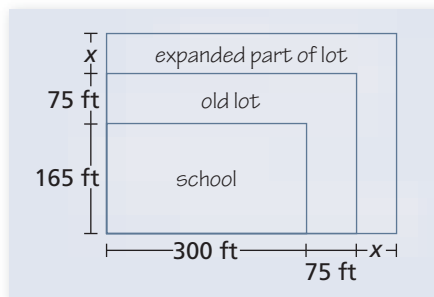
73. **DRAWING CONCLUSIONS** Consider the expression $x^2 + a^2$, where $a > 0$.

- You want to rewrite the expression as $(x + m)(x + n)$. Write two equations that m and n must satisfy.
- Use the equations you wrote in part (a) to solve for m and n . What can you conclude?

74. **THOUGHT PROVOKING** You are redesigning a rectangular raft. The raft is 6 feet long and 4 feet wide. You want to double the area of the raft by adding to the existing design. Draw a diagram of the new raft. Write and solve an equation you can use to find the dimensions of the new raft.



75. **MODELING WITH MATHEMATICS** A high school wants to double the size of its parking lot by expanding the existing lot as shown. By what distance x should the lot be expanded?



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the sum or difference. (*Skills Review Handbook*)

76. $(x^2 + 2) + (2x^2 - x)$

77. $(x^3 + x^2 - 4) + (3x^2 + 10)$

78. $(-2x + 1) - (-3x^2 + x)$

79. $(-3x^3 + x^2 - 12x) - (-6x^2 + 3x - 9)$

Find the product. (*Skills Review Handbook*)

80. $(x + 2)(x - 2)$

81. $2x(3 - x + 5x^2)$

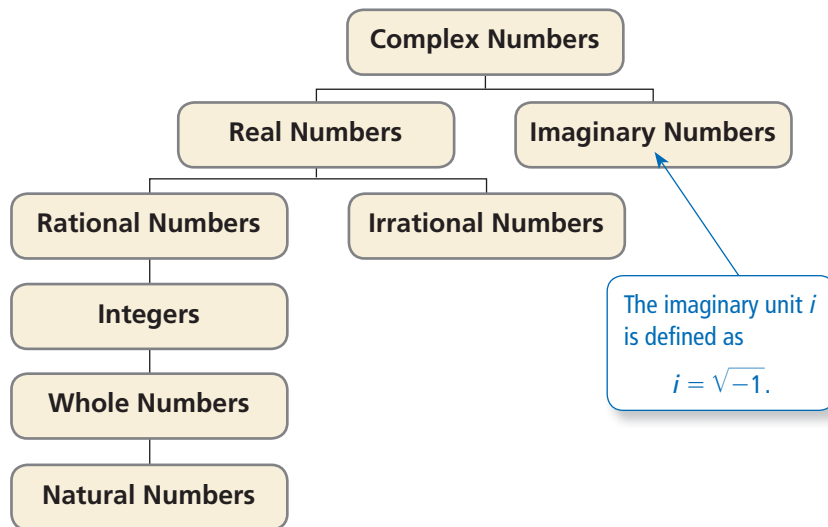
82. $(7 - x)(x - 1)$

83. $11x(-4x^2 + 3x + 8)$

3.2 Complex Numbers

Essential Question What are the subsets of the set of complex numbers?

In your study of mathematics, you have probably worked with only *real numbers*, which can be represented graphically on the real number line. In this lesson, the system of numbers is expanded to include *imaginary numbers*. The real numbers and imaginary numbers compose the set of *complex numbers*.



EXPLORATION 1 Classifying Numbers

Work with a partner. Determine which subsets of the set of complex numbers contain each number.

- | | | |
|-------------------------|---------------|----------------|
| a. $\sqrt{9}$ | b. $\sqrt{0}$ | c. $-\sqrt{4}$ |
| d. $\sqrt{\frac{4}{9}}$ | e. $\sqrt{2}$ | f. $\sqrt{-1}$ |

ATTENDING TO PRECISION

To be proficient in math, you need to use clear definitions in your reasoning and discussions with others.

EXPLORATION 2 Complex Solutions of Quadratic Equations

Work with a partner. Use the definition of the imaginary unit i to match each quadratic equation with its complex solution. Justify your answers.

- | | | |
|------------------|------------------|------------------|
| a. $x^2 - 4 = 0$ | b. $x^2 + 1 = 0$ | c. $x^2 - 1 = 0$ |
| d. $x^2 + 4 = 0$ | e. $x^2 - 9 = 0$ | f. $x^2 + 9 = 0$ |
| A. i | B. $3i$ | C. 3 |
| D. $2i$ | E. 1 | F. 2 |

Communicate Your Answer

- What are the subsets of the set of complex numbers? Give an example of a number in each subset.
- Is it possible for a number to be both whole and natural? natural and rational? rational and irrational? real and imaginary? Explain your reasoning.

3.2 Lesson

Core Vocabulary

imaginary unit i , p. 104
 complex number, p. 104
 imaginary number, p. 104
 pure imaginary number, p. 104

What You Will Learn

- ▶ Define and use the imaginary unit i .
- ▶ Add, subtract, and multiply complex numbers.
- ▶ Find complex solutions and zeros.

The Imaginary Unit i

Not all quadratic equations have real-number solutions. For example, $x^2 = -3$ has no real-number solutions because the square of any real number is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit i** , defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

Core Concept

The Square Root of a Negative Number

Property

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.

Example

$$\sqrt{-3} = i\sqrt{3}$$

$$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$$

EXAMPLE 1 Finding Square Roots of Negative Numbers

Find the square root of each number.

a. $\sqrt{-25}$ b. $\sqrt{-72}$ c. $-5\sqrt{-9}$

SOLUTION

a. $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$
 b. $\sqrt{-72} = \sqrt{72} \cdot \sqrt{-1} = \sqrt{36} \cdot \sqrt{2} \cdot i = 6\sqrt{2}i = 6i\sqrt{2}$
 c. $-5\sqrt{-9} = -5\sqrt{9} \cdot \sqrt{-1} = -5 \cdot 3 \cdot i = -15i$

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Find the square root of the number.

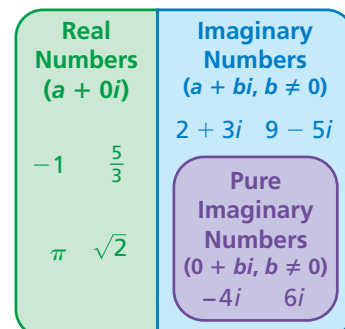
1. $\sqrt{-4}$ 2. $\sqrt{-12}$ 3. $-\sqrt{-36}$ 4. $2\sqrt{-54}$

A **complex number** written in *standard form* is a number $a + bi$ where a and b are real numbers. The number a is the *real part*, and the number bi is the *imaginary part*.

$$a + bi$$

If $b \neq 0$, then $a + bi$ is an **imaginary number**. If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number**. The diagram shows how different types of complex numbers are related.

Complex Numbers ($a + bi$)



Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$.

EXAMPLE 2 Equality of Two Complex Numbers

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

SOLUTION

Set the real parts equal to each other and the imaginary parts equal to each other.

$$2x = 10 \quad \text{Equate the real parts.} \quad -7i = yi \quad \text{Equate the imaginary parts.}$$

$$x = 5 \quad \text{Solve for } x. \quad -7 = y \quad \text{Solve for } y.$$

► So, $x = 5$ and $y = -7$.

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Find the values of x and y that satisfy the equation.

5. $x + 3i = 9 - yi$

6. $9 + 4yi = -2x + 3i$

Operations with Complex Numbers

Core Concept

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

EXAMPLE 3 Adding and Subtracting Complex Numbers

Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$

b. $(7 - 6i) - (3 - 6i)$

c. $13 - (2 + 7i) + 5i$

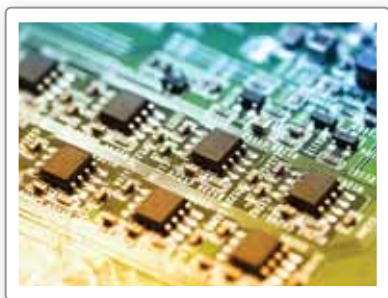
SOLUTION

a. $(8 - i) + (5 + 4i) = (8 + 5) + (-1 + 4)i$ Definition of complex addition
 $= 13 + 3i$ Write in standard form.




b. $(7 - 6i) - (3 - 6i) = (7 - 3) + (-6 + 6)i$ Definition of complex subtraction
 $= 4 + 0i$ Simplify.
 $= 4$ Write in standard form.

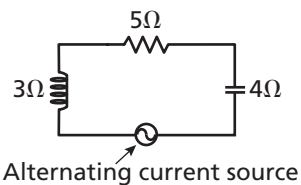
c. $13 - (2 + 7i) + 5i = [(13 - 2) - 7i] + 5i$ Definition of complex subtraction
 $= (11 - 7i) + 5i$ Simplify.
 $= 11 + (-7 + 5)i$ Definition of complex addition
 $= 11 - 2i$ Write in standard form.

EXAMPLE 4 Solving a Real-Life Problem



Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω , the uppercase Greek letter omega.

Component and symbol	Resistor 	Inductor 	Capacitor 
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	$-Ci$



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit.

SOLUTION

The resistor has a resistance of 5 ohms, so its impedance is 5 ohms. The inductor has a reactance of 3 ohms, so its impedance is $3i$ ohms. The capacitor has a reactance of 4 ohms, so its impedance is $-4i$ ohms.

$$\text{Impedance of circuit} = 5 + 3i + (-4i) = 5 - i$$

► The impedance of the circuit is $(5 - i)$ ohms.

To multiply two complex numbers, use the Distributive Property, or the FOIL method, just as you do when multiplying real numbers or algebraic expressions.

EXAMPLE 5 Multiplying Complex Numbers

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

b. $(9 - 2i)(-4 + 7i)$

SOLUTION

$$\begin{aligned} \text{a. } 4i(-6 + i) &= -24i + 4i^2 \\ &= -24i + 4(-1) \\ &= -4 - 24i \end{aligned}$$

Distributive Property

Use $i^2 = -1$.

Write in standard form.

$$\begin{aligned} \text{b. } (9 - 2i)(-4 + 7i) &= -36 + 63i + 8i - 14i^2 \\ &= -36 + 71i - 14(-1) \\ &= -36 + 71i + 14 \\ &= -22 + 71i \end{aligned}$$

Multiply using FOIL.

Simplify and use $i^2 = -1$.

Simplify.

Write in standard form.

STUDY TIP

When simplifying an expression that involves complex numbers, be sure to simplify i^2 as -1 .

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7. **WHAT IF?** In Example 4, what is the impedance of the circuit when the capacitor is replaced with one having a reactance of 7 ohms?

Perform the operation. Write the answer in standard form.

8. $(9 - i) + (-6 + 7i)$

9. $(3 + 7i) - (8 - 2i)$

10. $-4 - (1 + i) - (5 + 9i)$

11. $(-3i)(10i)$

12. $i(8 - i)$

13. $(3 + i)(5 - i)$

Complex Solutions and Zeros

EXAMPLE 6 Solving Quadratic Equations

Solve (a) $x^2 + 4 = 0$ and (b) $2x^2 - 11 = -47$.

SOLUTION

a. $x^2 + 4 = 0$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

Write original equation.

Subtract 4 from each side.

Take square root of each side.

Write in terms of i .

▶ The solutions are $2i$ and $-2i$.

b. $2x^2 - 11 = -47$

$$2x^2 = -36$$

$$x^2 = -18$$

$$x = \pm\sqrt{-18}$$

$$x = \pm i\sqrt{18}$$

$$x = \pm 3i\sqrt{2}$$

Write original equation.

Add 11 to each side.

Divide each side by 2.

Take square root of each side.

Write in terms of i .

Simplify radical.

▶ The solutions are $3i\sqrt{2}$ and $-3i\sqrt{2}$.

LOOKING FOR STRUCTURE

Notice that you can use the solutions in Example 6(a) to factor $x^2 + 4$ as $(x + 2i)(x - 2i)$.



EXAMPLE 7 Finding Zeros of a Quadratic Function

Find the zeros of $f(x) = 4x^2 + 20$.

SOLUTION

$$4x^2 + 20 = 0$$

$$4x^2 = -20$$

$$x^2 = -5$$

$$x = \pm\sqrt{-5}$$

$$x = \pm i\sqrt{5}$$

Set $f(x)$ equal to 0.

Subtract 20 from each side.

Divide each side by 4.

Take square root of each side.

Write in terms of i .

▶ So, the zeros of f are $i\sqrt{5}$ and $-i\sqrt{5}$.

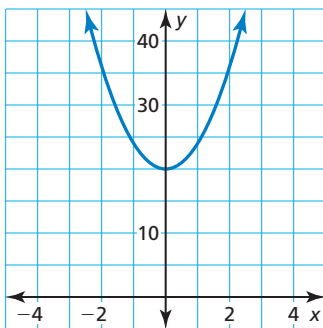
Check

$$f(i\sqrt{5}) = 4(i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0 \quad \checkmark$$

$$f(-i\sqrt{5}) = 4(-i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0 \quad \checkmark$$

FINDING AN ENTRY POINT

The graph of f does not intersect the x -axis, which means f has no real zeros. So, f must have complex zeros, which you can find algebraically.



Monitoring Progress



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Solve the equation.

14. $x^2 = -13$

15. $x^2 = -38$

16. $x^2 + 11 = 3$

17. $x^2 - 8 = -36$

18. $3x^2 - 7 = -31$

19. $5x^2 + 33 = 3$

Find the zeros of the function.

20. $f(x) = x^2 + 7$

21. $f(x) = -x^2 - 4$

22. $f(x) = 9x^2 + 1$

Vocabulary and Core Concept Check

- VOCABULARY** What is the imaginary unit i defined as and how can you use i ?
- COMPLETE THE SENTENCE** For the complex number $5 + 2i$, the imaginary part is ____ and the real part is ____.
- WRITING** Describe how to add complex numbers.
- WHICH ONE DOESN'T BELONG?** Which number does *not* belong with the other three? Explain your reasoning.

$3 + 0i$

$2 + 5i$

$\sqrt{3} + 6i$

$0 - 7i$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, find the square root of the number.
(See Example 1.)

- $\sqrt{-36}$
- $\sqrt{-64}$
- $\sqrt{-18}$
- $\sqrt{-24}$
- $2\sqrt{-16}$
- $-3\sqrt{-49}$
- $-4\sqrt{-32}$
- $6\sqrt{-63}$

In Exercises 13–20, find the values of x and y that satisfy the equation. (See Example 2.)

- $4x + 2i = 8 + yi$
- $3x + 6i = 27 + yi$
- $-10x + 12i = 20 + 3yi$
- $9x - 18i = -36 + 6yi$
- $2x - yi = 14 + 12i$
- $-12x + yi = 60 - 13i$
- $54 - \frac{1}{7}yi = 9x - 4i$
- $15 - 3yi = \frac{1}{2}x + 2i$

In Exercises 21–30, add or subtract. Write the answer in standard form. (See Example 3.)

- $(6 - i) + (7 + 3i)$
- $(9 + 5i) + (11 + 2i)$

$23. (12 + 4i) - (3 - 7i)$

$24. (2 - 15i) - (4 + 5i)$

$25. (12 - 3i) + (7 + 3i)$

$26. (16 - 9i) - (2 - 9i)$

$27. 7 - (3 + 4i) + 6i$

$28. 16 - (2 - 3i) - i$

$29. -10 + (6 - 5i) - 9i$

$30. -3 + (8 + 2i) + 7i$

31. USING STRUCTURE Write each expression as a complex number in standard form.

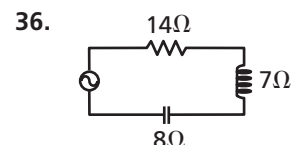
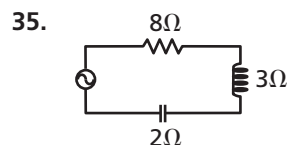
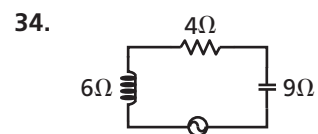
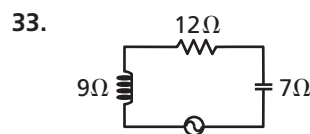
a. $\sqrt{-9} + \sqrt{-4} - \sqrt{16}$

b. $\sqrt{-16} + \sqrt{8} + \sqrt{-36}$

32. REASONING The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. Find the additive inverse of each complex number.

a. $z = 1 + i$ b. $z = 3 - i$ c. $z = -2 + 8i$

In Exercises 33–36, find the impedance of the series circuit. (See Example 4.)



In Exercises 37–44, multiply. Write the answer in standard form. (See Example 5.)

37. $3i(-5 + i)$ 38. $2i(7 - i)$
 39. $(3 - 2i)(4 + i)$ 40. $(7 + 5i)(8 - 6i)$
 41. $(4 - 2i)(4 + 2i)$ 42. $(9 + 5i)(9 - 5i)$
 43. $(3 - 6i)^2$ 44. $(8 + 3i)^2$

JUSTIFYING STEPS In Exercises 45 and 46, justify each step in performing the operation.

45. $11 - (4 + 3i) + 5i$
 $= [(11 - 4) - 3i] + 5i$
 $= (7 - 3i) + 5i$
 $= 7 + (-3 + 5)i$
 $= 7 + 2i$
46. $(3 + 2i)(7 - 4i)$
 $= 21 - 12i + 14i - 8i^2$
 $= 21 + 2i - 8(-1)$
 $= 21 + 2i + 8$
 $= 29 + 2i$

REASONING In Exercises 47 and 48, place the tiles in the expression to make a true statement.

47. $(\underline{\hspace{1cm}} - \underline{\hspace{1cm}}i) - (\underline{\hspace{1cm}} - \underline{\hspace{1cm}}i) = 2 - 4i$



48. $\underline{\hspace{1cm}}i(\underline{\hspace{1cm}} + \underline{\hspace{1cm}}i) = -18 - 10i$



In Exercises 49–54, solve the equation. Check your solution(s). (See Example 6.)

49. $x^2 + 9 = 0$ 50. $x^2 + 49 = 0$
 51. $x^2 - 4 = -11$
 52. $x^2 - 9 = -15$
 53. $2x^2 + 6 = -34$
 54. $x^2 + 7 = -47$

In Exercises 55–62, find the zeros of the function. (See Example 7.)

55. $f(x) = 3x^2 + 6$ 56. $g(x) = 7x^2 + 21$
 57. $h(x) = 2x^2 + 72$ 58. $k(x) = -5x^2 - 125$
 59. $m(x) = -x^2 - 27$ 60. $p(x) = x^2 + 98$
 61. $r(x) = -\frac{1}{2}x^2 - 24$ 62. $f(x) = -\frac{1}{5}x^2 - 10$

ERROR ANALYSIS In Exercises 63 and 64, describe and correct the error in performing the operation and writing the answer in standard form.

63. $(3 + 2i)(5 - i) = 15 - 3i + 10i - 2i^2$
 $= 15 + 7i - 2i^2$
 $= -2i^2 + 7i + 15$

64. $(4 + 6i)^2 = (4)^2 + (6i)^2$
 $= 16 + 36i^2$
 $= 16 + (36)(-1)$
 $= -20$

65. **NUMBER SENSE** Simplify each expression. Then classify your results in the table below.

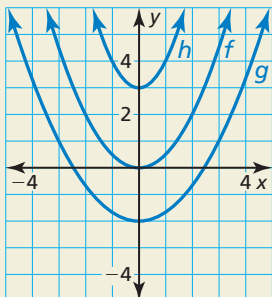
- a. $(-4 + 7i) + (-4 - 7i)$
 b. $(2 - 6i) - (-10 + 4i)$
 c. $(25 + 15i) - (25 - 6i)$
 d. $(5 + i)(8 - i)$
 e. $(17 - 3i) + (-17 - 6i)$
 f. $(-1 + 2i)(11 - i)$
 g. $(7 + 5i) + (7 - 5i)$
 h. $(-3 + 6i) - (-3 - 8i)$

Real numbers	Imaginary numbers	Pure imaginary numbers

66. **MAKING AN ARGUMENT** The Product Property of Square Roots states $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$. Your friend concludes $\sqrt{-4} \cdot \sqrt{-9} = \sqrt{36} = 6$. Is your friend correct? Explain.

67. **FINDING A PATTERN** Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. Describe the pattern you observe in the table. Verify the pattern continues by evaluating the next four powers of i .

68. **HOW DO YOU SEE IT?** The graphs of three functions are shown. Which function(s) has real zeros? imaginary zeros? Explain your reasoning.



In Exercises 69–74, write the expression as a complex number in standard form.

69. $(3 + 4i) - (7 - 5i) + 2i(9 + 12i)$
 70. $3i(2 + 5i) + (6 - 7i) - (9 + i)$
 71. $(3 + 5i)(2 - 7i^4)$
 72. $2i^3(5 - 12i)$
 73. $(2 + 4i^5) + (1 - 9i^6) - (3 + i^7)$
 74. $(8 - 2i^4) + (3 - 7i^8) - (4 + i^9)$

75. **OPEN-ENDED** Find two imaginary numbers whose sum and product are real numbers. How are the imaginary numbers related?
76. **COMPARING METHODS** Describe the two different methods shown for writing the complex expression in standard form. Which method do you prefer? Explain.

Method 1

$$\begin{aligned} 4i(2 - 3i) + 4i(1 - 2i) &= 8i - 12i^2 + 4i - 8i^2 \\ &= 8i - 12(-1) + 4i - 8(-1) \\ &= 20 + 12i \end{aligned}$$

Method 2

$$\begin{aligned} 4i(2 - 3i) + 4i(1 - 2i) &= 4i[(2 - 3i) + (1 - 2i)] \\ &= 4i[3 - 5i] \\ &= 12i - 20i^2 \\ &= 12i - 20(-1) \\ &= 20 + 12i \end{aligned}$$

77. **CRITICAL THINKING** Determine whether each statement is *true* or *false*. If it is true, give an example. If it is false, give a counterexample.
- The sum of two imaginary numbers is an imaginary number.
 - The product of two pure imaginary numbers is a real number.
 - A pure imaginary number is an imaginary number.
 - A complex number is a real number.
78. **THOUGHT PROVOKING** Create a circuit that has an impedance of $14 - 3i$.

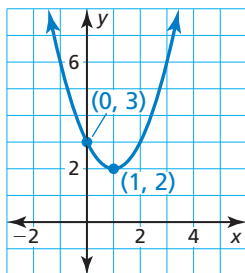
Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Determine whether the given value of x is a solution to the equation. (*Skills Review Handbook*)

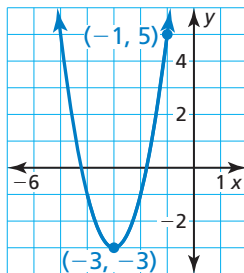
79. $3(x - 2) + 4x - 1 = x - 1$; $x = 1$ 80. $x^3 - 6 = 2x^2 + 9 - 3x$; $x = -5$ 81. $-x^2 + 4x = \frac{19}{3}x^2$; $x = -\frac{3}{4}$

Write a quadratic function in vertex form whose graph is shown. (*Section 2.4*)

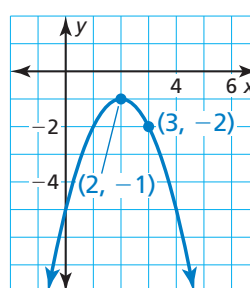
82.



83.



84.



3.3 Completing the Square

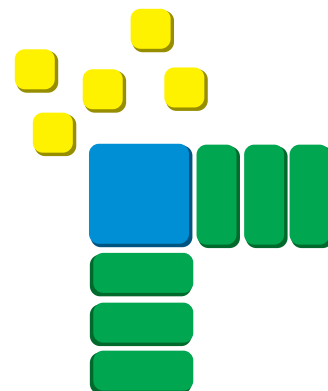
Essential Question How can you complete the square for a quadratic expression?

EXPLORATION 1 Using Algebra Tiles to Complete the Square

Work with a partner. Use algebra tiles to complete the square for the expression

$$x^2 + 6x.$$

- You can model $x^2 + 6x$ using one x^2 -tile and six x -tiles. Arrange the tiles in a square. Your arrangement will be incomplete in one of the corners.
- How many 1-tiles do you need to complete the square?
- Find the value of c so that the expression $x^2 + 6x + c$ is a perfect square trinomial.
- Write the expression in part (c) as the square of a binomial.



EXPLORATION 2 Drawing Conclusions

Work with a partner.

- Use the method outlined in Exploration 1 to complete the table.

Expression	Value of c needed to complete the square	Expression written as a binomial squared
$x^2 + 2x + c$		
$x^2 + 4x + c$		
$x^2 + 8x + c$		
$x^2 + 10x + c$		

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

- Look for patterns in the last column of the table. Consider the general statement $x^2 + bx + c = (x + d)^2$. How are d and b related in each case? How are c and d related in each case?
- How can you obtain the values in the second column directly from the coefficients of x in the first column?

Communicate Your Answer

- How can you complete the square for a quadratic expression?
- Describe how you can solve the quadratic equation $x^2 + 6x = 1$ by completing the square.

3.3 Lesson

Core Vocabulary

completing the square, p. 112

Previous

perfect square trinomial
vertex form

ANOTHER WAY

You can also solve the equation by writing it in standard form as $x^2 - 16x - 36 = 0$ and factoring.



What You Will Learn

- ▶ Solve quadratic equations using square roots.
- ▶ Solve quadratic equations by completing the square.
- ▶ Write quadratic functions in vertex form.

Solving Quadratic Equations Using Square Roots

Previously, you have solved equations of the form $u^2 = d$ by taking the square root of each side. This method also works when one side of an equation is a perfect square trinomial and the other side is a constant.

EXAMPLE 1 Solving a Quadratic Equation Using Square Roots

Solve $x^2 - 16x + 64 = 100$ using square roots.

SOLUTION

$$x^2 - 16x + 64 = 100$$

Write the equation.

$$(x - 8)^2 = 100$$

Write the left side as a binomial squared.

$$x - 8 = \pm 10$$

Take square root of each side.

$$x = 8 \pm 10$$

Add 8 to each side.

- ▶ So, the solutions are $x = 8 + 10 = 18$ and $x = 8 - 10 = -2$.

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Solve the equation using square roots. Check your solution(s).

1. $x^2 + 4x + 4 = 36$
2. $x^2 - 6x + 9 = 1$
3. $x^2 - 22x + 121 = 81$

In Example 1, the expression $x^2 - 16x + 64$ is a perfect square trinomial because it equals $(x - 8)^2$. Sometimes you need to add a term to an expression $x^2 + bx$ to make it a perfect square trinomial. This process is called **completing the square**.

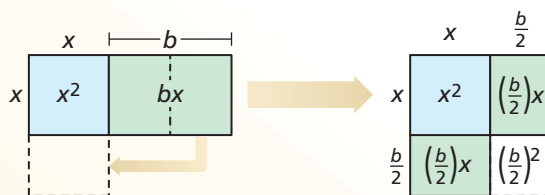
Core Concept

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

Diagrams In each diagram, the combined area of the shaded regions is $x^2 + bx$.

Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Solving Quadratic Equations by Completing the Square

EXAMPLE 2 Making a Perfect Square Trinomial

Find the value of c that makes $x^2 + 14x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

SOLUTION

Step 1 Find half the coefficient of x .

$$\frac{14}{2} = 7$$

Step 2 Square the result of Step 1.

$$7^2 = 49$$

Step 3 Replace c with the result of Step 2.

$$x^2 + 14x + 49$$

	x	7
x	x^2	$7x$
7	$7x$	49

► The expression $x^2 + 14x + c$ is a perfect square trinomial when $c = 49$. Then $x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$.

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Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

4. $x^2 + 8x + c$

5. $x^2 - 2x + c$

6. $x^2 - 9x + c$

The method of completing the square can be used to solve *any* quadratic equation. When you complete the square as part of solving an equation, you must add the same number to *both* sides of the equation.

LOOKING FOR STRUCTURE

Notice you cannot solve the equation by factoring because $x^2 - 10x + 7$ is not factorable as a product of binomials.

EXAMPLE 3 Solving $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 10x + 7 = 0$ by completing the square.

SOLUTION

$$x^2 - 10x + 7 = 0$$

Write the equation.

$$x^2 - 10x = -7$$

Write left side in the form $x^2 + bx$.

$$x^2 - 10x + 25 = -7 + 25$$

Add $\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = 25$ to each side.

$$(x - 5)^2 = 18$$

Write left side as a binomial squared.

$$x - 5 = \pm\sqrt{18}$$

Take square root of each side.

$$x = 5 \pm\sqrt{18}$$

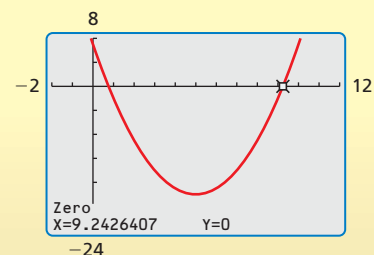
Add 5 to each side.

$$x = 5 \pm 3\sqrt{2}$$

Simplify radical.

► The solutions are $x = 5 + 3\sqrt{2}$ and $x = 5 - 3\sqrt{2}$. You can check this by graphing $y = x^2 - 10x + 7$. The x -intercepts are about $9.24 \approx 5 + 3\sqrt{2}$ and $0.76 \approx 5 - 3\sqrt{2}$.

Check



EXAMPLE 4 Solving $ax^2 + bx + c = 0$ when $a \neq 1$ Solve $3x^2 + 12x + 15 = 0$ by completing the square.**SOLUTION**The coefficient a is not 1, so you must first divide each side of the equation by a .

$$3x^2 + 12x + 15 = 0$$

Write the equation.

$$x^2 + 4x + 5 = 0$$

Divide each side by 3.

$$x^2 + 4x = -5$$

Write left side in the form $x^2 + bx$.

$$x^2 + 4x + 4 = -5 + 4$$

Add $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$ to each side.

$$(x + 2)^2 = -1$$

Write left side as a binomial squared.

$$x + 2 = \pm\sqrt{-1}$$

Take square root of each side.

$$x = -2 \pm \sqrt{-1}$$

Subtract 2 from each side.

$$x = -2 \pm i$$

Write in terms of i .▶ The solutions are $x = -2 + i$ and $x = -2 - i$.**Monitoring Progress**Help in English and Spanish at BigIdeasMath.com

Solve the equation by completing the square.

7. $x^2 - 4x + 8 = 0$

8. $x^2 + 8x - 5 = 0$

9. $-3x^2 - 18x - 6 = 0$

10. $4x^2 + 32x = -68$

11. $6x(x + 2) = -42$

12. $2x(x - 2) = 200$

Writing Quadratic Functions in Vertex FormRecall that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex of the graph of the function. You can write a quadratic function in vertex form by completing the square.**EXAMPLE 5** Writing a Quadratic Function in Vertex FormWrite $y = x^2 - 12x + 18$ in vertex form. Then identify the vertex.**SOLUTION**

$$y = x^2 - 12x + 18$$

Write the function.

$$y + ? = (x^2 - 12x + ?) + 18$$

Prepare to complete the square.

$$y + 36 = (x^2 - 12x + 36) + 18$$

Add $\left(\frac{b}{2}\right)^2 = \left(\frac{-12}{2}\right)^2 = 36$ to each side.

$$y + 36 = (x - 6)^2 + 18$$

Write $x^2 - 12x + 36$ as a binomial squared.

$$y = (x - 6)^2 - 18$$

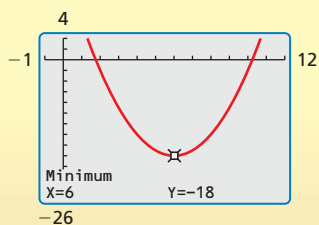
Solve for y .▶ The vertex form of the function is $y = (x - 6)^2 - 18$. The vertex is $(6, -18)$.**Monitoring Progress**Help in English and Spanish at BigIdeasMath.com

Write the quadratic function in vertex form. Then identify the vertex.

13. $y = x^2 - 8x + 18$

14. $y = x^2 + 6x + 4$

15. $y = x^2 - 2x - 6$

Check

EXAMPLE 6

Modeling with Mathematics

The height y (in feet) of a baseball t seconds after it is hit can be modeled by the function

$$y = -16t^2 + 96t + 3.$$

Find the maximum height of the baseball.
How long does the ball take to hit the ground?



SOLUTION

- 1. Understand the Problem** You are given a quadratic function that represents the height of a ball. You are asked to determine the maximum height of the ball and how long it is in the air.
- 2. Make a Plan** Write the function in vertex form to identify the maximum height. Then find and interpret the zeros to determine how long the ball takes to hit the ground.

- 3. Solve the Problem** Write the function in vertex form by completing the square.

$$y = -16t^2 + 96t + 3$$

Write the function.

$$y = -16(t^2 - 6t) + 3$$

Factor -16 from first two terms.

$$y + ? = -16(t^2 - 6t + ?) + 3$$

Prepare to complete the square.

$$y + (-16)(9) = -16(t^2 - 6t + 9) + 3$$

Add $(-16)(9)$ to each side.

$$y - 144 = -16(t - 3)^2 + 3$$

Write $t^2 - 6t + 9$ as a binomial squared.

$$y = -16(t - 3)^2 + 147$$

Solve for y .

The vertex is $(3, 147)$. Find the zeros of the function.

$$0 = -16(t - 3)^2 + 147$$

Substitute 0 for y .

$$-147 = -16(t - 3)^2$$

Subtract 147 from each side.

$$9.1875 = (t - 3)^2$$

Divide each side by -16 .

$$\pm\sqrt{9.1875} = t - 3$$

Take square root of each side.

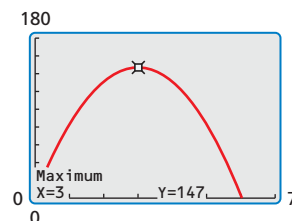
$$3 \pm \sqrt{9.1875} = t$$

Add 3 to each side.

Reject the negative solution, $3 - \sqrt{9.1875} \approx -0.03$, because time must be positive.

- So, the maximum height of the ball is 147 feet, and it takes $3 + \sqrt{9.1875} \approx 6$ seconds for the ball to hit the ground.

- 4. Look Back** The vertex indicates that the maximum height of 147 feet occurs when $t = 3$. This makes sense because the graph of the function is parabolic with zeros near $t = 0$ and $t = 6$. You can use a graph to check the maximum height.



ANOTHER WAY

You can use the coefficients of the original function $y = f(x)$ to find the maximum height.

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= f\left(-\frac{96}{2(-16)}\right) \\ &= f(3) \\ &= 147 \end{aligned}$$

LOOKING FOR STRUCTURE

You could write the zeros as $3 \pm \frac{7\sqrt{3}}{4}$, but it is easier to recognize that $3 - \sqrt{9.1875}$ is negative because $\sqrt{9.1875}$ is greater than 3.

Monitoring Progress



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- 16. WHAT IF?** The height of the baseball can be modeled by $y = -16t^2 + 80t + 2$. Find the maximum height of the baseball. How long does the ball take to hit the ground?

3.3 Exercises

Vocabulary and Core Concept Check

- VOCABULARY** What must you add to the expression $x^2 + bx$ to complete the square?
- COMPLETE THE SENTENCE** The trinomial $x^2 - 6x + 9$ is a _____ because it equals _____.

Monitoring Progress and Modeling with Mathematics

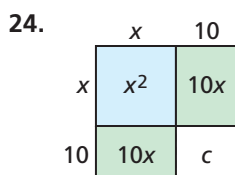
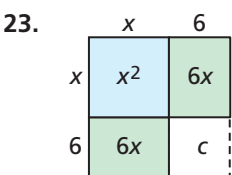
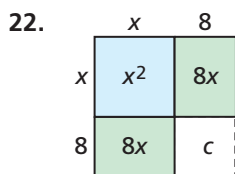
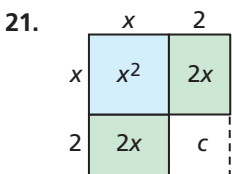
In Exercises 3–10, solve the equation using square roots. Check your solution(s). (See Example 1.)

- $x^2 - 8x + 16 = 25$
- $r^2 - 10r + 25 = 1$
- $x^2 - 18x + 81 = 5$
- $m^2 + 8m + 16 = 45$
- $y^2 - 24y + 144 = -100$
- $x^2 - 26x + 169 = -13$
- $4w^2 + 4w + 1 = 75$
- $4x^2 - 8x + 4 = 1$

In Exercises 11–20, find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial. (See Example 2.)

- $x^2 + 10x + c$
- $x^2 + 20x + c$
- $y^2 - 12y + c$
- $t^2 - 22t + c$
- $x^2 - 6x + c$
- $x^2 + 24x + c$
- $z^2 - 5z + c$
- $x^2 + 9x + c$
- $w^2 + 13w + c$
- $s^2 - 26s + c$

In Exercises 21–24, find the value of c . Then write an expression represented by the diagram.



In Exercises 25–36, solve the equation by completing the square. (See Examples 3 and 4.)

- $x^2 + 6x + 3 = 0$
- $s^2 + 2s - 6 = 0$
- $x^2 + 4x - 2 = 0$
- $t^2 - 8t - 5 = 0$
- $z(z + 9) = 1$
- $x(x + 8) = -20$
- $7t^2 + 28t + 56 = 0$
- $6r^2 + 6r + 12 = 0$
- $5x(x + 6) = -50$
- $4w(w - 3) = 24$
- $4x^2 - 30x = 12 + 10x$
- $3s^2 + 8s = 2s - 9$
- ERROR ANALYSIS** Describe and correct the error in solving the equation.

X

$$4x^2 + 24x - 11 = 0$$

$$4(x^2 + 6x) = 11$$

$$4(x^2 + 6x + 9) = 11 + 9$$

$$4(x + 3)^2 = 20$$

$$(x + 3)^2 = 5$$

$$x + 3 = \pm\sqrt{5}$$

$$x = -3 \pm\sqrt{5}$$

- ERROR ANALYSIS** Describe and correct the error in finding the value of c that makes the expression a perfect square trinomial.

X

$$x^2 + 30x + c$$

$$x^2 + 30x + \frac{30}{2}$$

$$x^2 + 30x + 15$$

- WRITING** Can you solve an equation by completing the square when the equation has two imaginary solutions? Explain.

40. **ABSTRACT REASONING** Which of the following are solutions of the equation $x^2 - 2ax + a^2 = b^2$? Justify your answers.

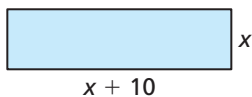
- (A) ab (B) $-a - b$
 (C) b (D) a
 (E) $a - b$ (F) $a + b$

USING STRUCTURE In Exercises 41–50, determine whether you would use factoring, square roots, or completing the square to solve the equation. Explain your reasoning. Then solve the equation.

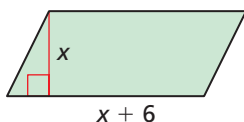
41. $x^2 - 4x - 21 = 0$ 42. $x^2 + 13x + 22 = 0$
 43. $(x + 4)^2 = 16$ 44. $(x - 7)^2 = 9$
 45. $x^2 + 12x + 36 = 0$
 46. $x^2 - 16x + 64 = 0$
 47. $2x^2 + 4x - 3 = 0$
 48. $3x^2 + 12x + 1 = 0$
 49. $x^2 - 100 = 0$ 50. $4x^2 - 20 = 0$

MATHEMATICAL CONNECTIONS In Exercises 51–54, find the value of x .

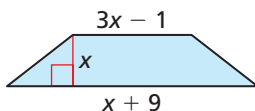
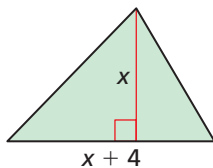
51. Area of rectangle = 50



52. Area of parallelogram = 48



53. Area of triangle = 40 54. Area of trapezoid = 20



In Exercises 55–62, write the quadratic function in vertex form. Then identify the vertex. (See Example 5.)

55. $f(x) = x^2 - 8x + 19$
 56. $g(x) = x^2 - 4x - 1$
 57. $g(x) = x^2 + 12x + 37$
 58. $h(x) = x^2 + 20x + 90$
 59. $h(x) = x^2 + 2x - 48$

60. $f(x) = x^2 + 6x - 16$

61. $f(x) = x^2 - 3x + 4$

62. $g(x) = x^2 + 7x + 2$

63. **MODELING WITH MATHEMATICS** While marching, a drum major tosses a baton into the air and catches it. The height h (in feet) of the baton t seconds after it is thrown can be modeled by the function $h = -16t^2 + 32t + 6$. (See Example 6.)

- a. Find the maximum height of the baton.
 b. The drum major catches the baton when it is 4 feet above the ground. How long is the baton in the air?

64. **MODELING WITH MATHEMATICS** A firework explodes when it reaches its maximum height. The height h (in feet) of the firework t seconds after it is launched can be modeled by $h = -\frac{500}{9}t^2 + \frac{1000}{3}t + 10$. What is the maximum height of the firework? How long is the firework in the air before it explodes?

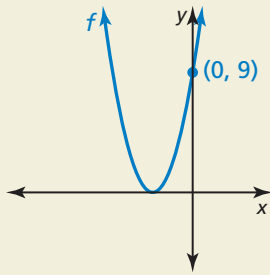


65. **COMPARING METHODS** A skateboard shop sells about 50 skateboards per week when the advertised price is charged. For each \$1 decrease in price, one additional skateboard per week is sold. The shop's revenue can be modeled by $y = (70 - x)(50 + x)$.



- a. Use the intercept form of the function to find the maximum weekly revenue.
 b. Write the function in vertex form to find the maximum weekly revenue.
 c. Which way do you prefer? Explain your reasoning.

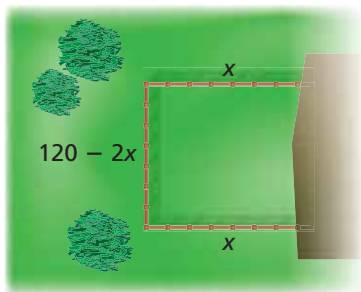
66. **HOW DO YOU SEE IT?** The graph of the function $f(x) = (x - h)^2$ is shown. What is the x -intercept? Explain your reasoning.



67. **WRITING** At Buckingham Fountain in Chicago, the height h (in feet) of the water above the main nozzle can be modeled by $h = -16t^2 + 89.6t$, where t is the time (in seconds) since the water has left the nozzle. Describe three different ways you could find the maximum height the water reaches. Then choose a method and find the maximum height of the water.

68. **PROBLEM SOLVING** A farmer is building a rectangular pen along the side of a barn for animals. The barn will serve as one side of the pen. The farmer has 120 feet of fence to enclose an area of 1512 square feet and wants each side of the pen to be at least 20 feet long.

- Write an equation that represents the area of the pen.
- Solve the equation in part (a) to find the dimensions of the pen.



69. **MAKING AN ARGUMENT** Your friend says the equation $x^2 + 10x = -20$ can be solved by either completing the square or factoring. Is your friend correct? Explain.

70. **THOUGHT PROVOKING** Write a function g in standard form whose graph has the same x -intercepts as the graph of $f(x) = 2x^2 + 8x + 2$. Find the zeros of each function by completing the square. Graph each function.

71. **CRITICAL THINKING** Solve $x^2 + bx + c = 0$ by completing the square. Your answer will be an expression for x in terms of b and c .

72. **DRAWING CONCLUSIONS** In this exercise, you will investigate the graphical effect of completing the square.

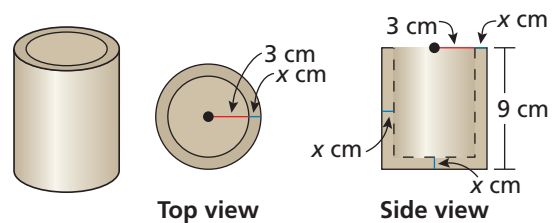
- Graph each pair of functions in the same coordinate plane.

$$\begin{array}{ll} y = x^2 + 2x & y = x^2 - 6x \\ y = (x + 1)^2 & y = (x - 3)^2 \end{array}$$

- Compare the graphs of $y = x^2 + bx$ and

$$y = \left(x + \frac{b}{2}\right)^2.$$
 Describe what happens to the graph of $y = x^2 + bx$ when you complete the square.

73. **MODELING WITH MATHEMATICS** In your pottery class, you are given a lump of clay with a volume of 200 cubic centimeters and are asked to make a cylindrical pencil holder. The pencil holder should be 9 centimeters high and have an inner radius of 3 centimeters. What thickness x should your pencil holder have if you want to use all of the clay?



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. (*Skills Review Handbook*)

74. $2x - 3 < 5$

75. $4 - 8y \geq 12$

76. $\frac{n}{3} + 6 > 1$

77. $-\frac{2s}{5} \leq 8$

Graph the function. Label the vertex, axis of symmetry, and x -intercepts. (*Section 2.2*)

78. $g(x) = 6(x - 4)^2$

79. $h(x) = 2x(x - 3)$

80. $f(x) = x^2 + 2x + 5$

81. $f(x) = 2(x + 10)(x - 12)$

3.1–3.3 What Did You Learn?

Core Vocabulary

quadratic equation in one variable, *p.* 94
root of an equation, *p.* 94
zero of a function, *p.* 96
imaginary unit i , *p.* 104

complex number, *p.* 104
imaginary number, *p.* 104
pure imaginary number, *p.* 104
completing the square, *p.* 112

Core Concepts

Section 3.1

Solving Quadratic Equations Graphically, *p.* 94
Solving Quadratic Equations Algebraically, *p.* 95

Zero-Product Property, *p.* 96

Section 3.2

The Square Root of a Negative Number, *p.* 104

Operations with Complex Numbers, *p.* 105

Section 3.3

Solving Quadratic Equations by Completing the Square, *p.* 113

Writing Quadratic Functions in Vertex Form, *p.* 114

Mathematical Practices

1. Analyze the givens, constraints, relationships, and goals in Exercise 61 on page 101.
2. Determine whether it would be easier to find the zeros of the function in Exercise 63 on page 117 or Exercise 67 on page 118.

Study Skills

Creating a Positive Study Environment

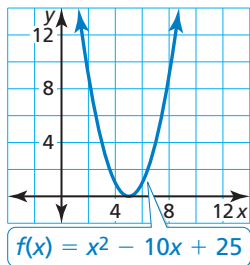
- Set aside an appropriate amount of time for reviewing your notes and the textbook, reworking your notes, and completing homework.
- Set up a place for studying at home that is comfortable, but not too comfortable. The place needs to be away from all potential distractions.
- Form a study group. Choose students who study well together, help out when someone misses school, and encourage positive attitudes.



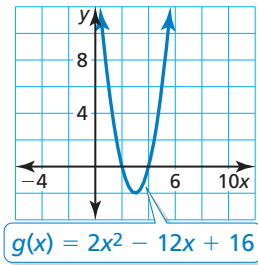
3.1–3.3 Quiz

Solve the equation by using the graph. Check your solution(s). (Section 3.1)

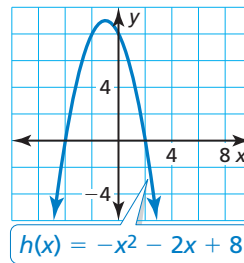
1. $x^2 - 10x + 25 = 0$



2. $2x^2 + 16 = 12x$



3. $x^2 = -2x + 8$



Solve the equation using square roots or by factoring. Explain the reason for your choice. (Section 3.1)

4. $2x^2 - 15 = 0$

5. $3x^2 - x - 2 = 0$

6. $(x + 3)^2 = 8$

7. Find the values of x and y that satisfy the equation $7x - 6i = 14 + yi$. (Section 3.2)

Perform the operation. Write your answer in standard form. (Section 3.2)

8. $(2 + 5i) + (-4 + 3i)$

9. $(3 + 9i) - (1 - 7i)$

10. $(2 + 4i)(-3 - 5i)$

11. Find the zeros of the function $f(x) = 9x^2 + 2$. Does the graph of the function intersect the x -axis? Explain your reasoning. (Section 3.2)

Solve the equation by completing the square. (Section 3.3)

12. $x^2 - 6x + 10 = 0$

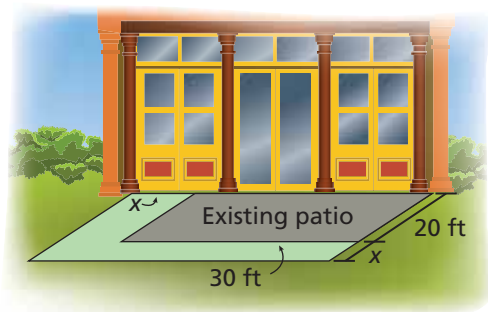
13. $x^2 + 12x + 4 = 0$

14. $4x(x + 6) = -40$

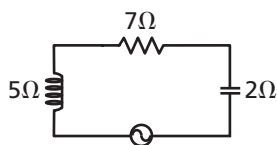
15. Write $y = x^2 - 10x + 4$ in vertex form. Then identify the vertex. (Section 3.3)

16. A museum has a café with a rectangular patio. The museum wants to add 464 square feet to the area of the patio by expanding the existing patio as shown. (Section 3.1)

- Find the area of the existing patio.
- Write an equation to model the area of the new patio.
- By what distance x should the length of the patio be expanded?



17. Find the impedance of the series circuit. (Section 3.2)



18. The height h (in feet) of a badminton birdie t seconds after it is hit can be modeled by the function $h = -16t^2 + 32t + 4$. (Section 3.3)

- Find the maximum height of the birdie.
- How long is the birdie in the air?

3.4 Using the Quadratic Formula

Essential Question How can you derive a general formula for solving a quadratic equation?

EXPLORATION 1 Deriving the Quadratic Formula

Work with a partner. Analyze and describe what is done in each step in the development of the Quadratic Formula.

REASONING ABSTRACTLY

To be proficient in math, you need to create a coherent representation of the problem at hand.

Step	Justification
$ax^2 + bx + c = 0$	
$ax^2 + bx = -c$	
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2 a }$	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

The result is the Quadratic Formula.

EXPLORATION 2 Using the Quadratic Formula

Work with a partner. Use the Quadratic Formula to solve each equation.

- | | |
|------------------------|-----------------------|
| a. $x^2 - 4x + 3 = 0$ | b. $x^2 - 2x + 2 = 0$ |
| c. $x^2 + 2x - 3 = 0$ | d. $x^2 + 4x + 4 = 0$ |
| e. $x^2 - 6x + 10 = 0$ | f. $x^2 + 4x + 6 = 0$ |

Communicate Your Answer

- How can you derive a general formula for solving a quadratic equation?
- Summarize the following methods you have learned for solving quadratic equations: graphing, using square roots, factoring, completing the square, and using the Quadratic Formula.

3.4 Lesson

Core Vocabulary

Quadratic Formula, p. 122
discriminant, p. 124

What You Will Learn

- ▶ Solve quadratic equations using the Quadratic Formula.
- ▶ Analyze the discriminant to determine the number and type of solutions.
- ▶ Solve real-life problems.

Solving Equations Using the Quadratic Formula

Previously, you solved quadratic equations by completing the square. In the Exploration, you developed a formula that gives the solutions of any quadratic equation by completing the square once for the general equation $ax^2 + bx + c = 0$. The formula for the solutions is called the **Quadratic Formula**.

Core Concept

The Quadratic Formula

Let a , b , and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

COMMON ERROR

Remember to write the quadratic equation in standard form before applying the Quadratic Formula.

EXAMPLE 1 Solving an Equation with Two Real Solutions

Solve $x^2 + 3x = 5$ using the Quadratic Formula.

SOLUTION

$$x^2 + 3x = 5$$

Write original equation.

$$x^2 + 3x - 5 = 0$$

Write in standard form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$

Substitute 1 for a , 3 for b , and -5 for c .

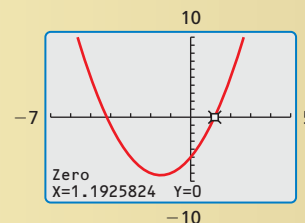
$$x = \frac{-3 \pm \sqrt{29}}{2}$$

Simplify.

▶ So, the solutions are $x = \frac{-3 + \sqrt{29}}{2} \approx 1.19$ and $x = \frac{-3 - \sqrt{29}}{2} \approx -4.19$.

Check Graph $y = x^2 + 3x - 5$.

The x -intercepts are about -4.19 and about 1.19 . ✓



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Solve the equation using the Quadratic Formula.

1. $x^2 - 6x + 4 = 0$

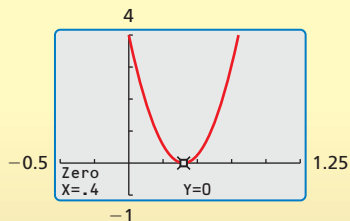
2. $2x^2 + 4 = -7x$

3. $5x^2 = x + 8$

ANOTHER WAY

You can also use factoring to solve $25x^2 - 20x + 4 = 0$ because the left side factors as $(5x - 2)^2$.

Check



EXAMPLE 2 Solving an Equation with One Real Solution

Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

SOLUTION

$$25x^2 - 8x = 12x - 4$$

Write original equation.

$$25x^2 - 20x + 4 = 0$$

Write in standard form.

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$

$$a = 25, b = -20, c = 4$$

$$x = \frac{20 \pm \sqrt{0}}{50}$$

Simplify.

$$x = \frac{2}{5}$$

Simplify.

► So, the solution is $x = \frac{2}{5}$. You can check this by graphing $y = 25x^2 - 20x + 4$. The only x -intercept is $\frac{2}{5}$.

EXAMPLE 3 Solving an Equation with Imaginary Solutions

Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

SOLUTION

$$-x^2 + 4x = 13$$

Write original equation.

$$-x^2 + 4x - 13 = 0$$

Write in standard form.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-13)}}{2(-1)}$$

$$a = -1, b = 4, c = -13$$

$$x = \frac{-4 \pm \sqrt{-36}}{-2}$$

Simplify.

$$x = \frac{-4 \pm 6i}{-2}$$

Write in terms of i .

$$x = 2 \pm 3i$$

Simplify.

► The solutions are $x = 2 + 3i$ and $x = 2 - 3i$.

COMMON ERROR

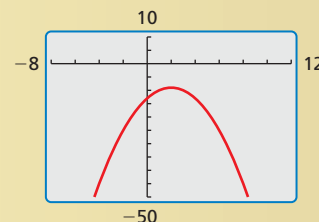
Remember to divide the real part *and* the imaginary part by -2 when simplifying.

Check Graph $y = -x^2 + 4x - 13$. There are no x -intercepts. So, the original equation has no real solutions. The algebraic check for one of the imaginary solutions is shown.

$$-(2 + 3i)^2 + 4(2 + 3i) \stackrel{?}{=} 13$$

$$5 - 12i + 8 + 12i \stackrel{?}{=} 13$$

$$13 = 13 \quad \checkmark$$



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Solve the equation using the Quadratic Formula.

4. $x^2 + 41 = -8x$

5. $-9x^2 = 30x + 25$

6. $5x - 7x^2 = 3x + 4$

Analyzing the Discriminant

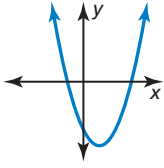
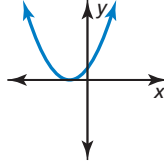
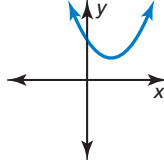
In the Quadratic Formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

You can analyze the discriminant of a quadratic equation to determine the number and type of solutions of the equation.

Core Concept

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

EXAMPLE 4 Analyzing the Discriminant

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$

b. $x^2 - 6x + 9 = 0$

c. $x^2 - 6x + 8 = 0$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a. $x^2 - 6x + 10 = 0$	$(-6)^2 - 4(1)(10) = -4$	Two imaginary: $3 \pm i$
b. $x^2 - 6x + 9 = 0$	$(-6)^2 - 4(1)(9) = 0$	One real: 3
c. $x^2 - 6x + 8 = 0$	$(-6)^2 - 4(1)(8) = 4$	Two real: 2, 4

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Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

7. $4x^2 + 8x + 4 = 0$

8. $\frac{1}{2}x^2 + x - 1 = 0$

9. $5x^2 = 8x - 13$

10. $7x^2 - 3x = 6$

11. $4x^2 + 6x = -9$

12. $-5x^2 + 1 = 6 - 10x$

EXAMPLE 5 Writing an Equation

Find a possible pair of integer values for a and c so that the equation $ax^2 - 4x + c = 0$ has one real solution. Then write the equation.

SOLUTION

In order for the equation to have one real solution, the discriminant must equal 0.

$$b^2 - 4ac = 0 \quad \text{Write the discriminant.}$$

$$(-4)^2 - 4ac = 0 \quad \text{Substitute } -4 \text{ for } b.$$

$$16 - 4ac = 0 \quad \text{Evaluate the power.}$$

$$-4ac = -16 \quad \text{Subtract 16 from each side.}$$

$$ac = 4 \quad \text{Divide each side by } -4.$$

Because $ac = 4$, choose two integers whose product is 4, such as $a = 1$ and $c = 4$.

► So, one possible equation is $x^2 - 4x + 4 = 0$.

ANOTHER WAY

Another possible equation in Example 5 is $4x^2 - 4x + 1 = 0$. You can obtain this equation by letting $a = 4$ and $c = 1$.

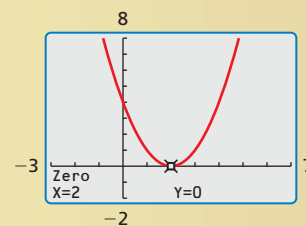


Check Graph $y = x^2 - 4x + 4$. The only x -intercept is 2. You can also check by factoring.

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x = 2 \quad \checkmark$$



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13. Find a possible pair of integer values for a and c so that the equation $ax^2 + 3x + c = 0$ has two real solutions. Then write the equation.

The table shows five methods for solving quadratic equations. For a given equation, it may be more efficient to use one method instead of another. Suggestions about when to use each method are shown below.

Concept Summary

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for <i>any</i> quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for <i>any</i> quadratic equation.

Solving Real-Life Problems

The function $h = -16t^2 + h_0$ is used to model the height of a *dropped* object. For an object that is *launched* or *thrown*, an extra term v_0t must be added to the model to account for the object's initial vertical velocity v_0 (in feet per second). Recall that h is the height (in feet), t is the time in motion (in seconds), and h_0 is the initial height (in feet).

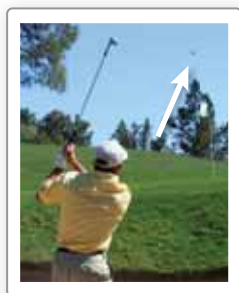
$$h = -16t^2 + h_0$$

Object is dropped.

$$h = -16t^2 + v_0t + h_0$$

Object is launched or thrown.

As shown below, the value of v_0 can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.



$$V_0 > 0$$



$$V_0 < 0$$



$$V_0 = 0$$

EXAMPLE 6

Modeling a Launched Object

A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

SOLUTION

Because the ball is *thrown*, use the model $h = -16t^2 + v_0t + h_0$. To find how long the ball is in the air, solve for t when $h = 3$.

$$h = -16t^2 + v_0t + h_0$$

Write the height model.

$$3 = -16t^2 + 30t + 4$$

Substitute 3 for h , 30 for v_0 , and 4 for h_0 .

$$0 = -16t^2 + 30t + 1$$

Write in standard form.

This equation is not factorable, and completing the square would result in fractions. So, use the Quadratic Formula to solve the equation.

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(1)}}{2(-16)}$$

$$a = -16, b = 30, c = 1$$

$$t = \frac{-30 \pm \sqrt{964}}{-32}$$

Simplify.

$$t \approx -0.033 \text{ or } t \approx 1.9$$

Use a calculator.

- Reject the negative solution, -0.033 , because the ball's time in the air cannot be negative. So, the ball is in the air for about 1.9 seconds.

Monitoring Progress



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14. **WHAT IF?** The ball leaves the juggler's hand with an initial vertical velocity of 40 feet per second. How long is the ball in the air?

3.4 Exercises

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** When a , b , and c are real numbers such that $a \neq 0$, the solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \underline{\hspace{2cm}}$.
- COMPLETE THE SENTENCE** You can use the $\underline{\hspace{2cm}}$ of a quadratic equation to determine the number and type of solutions of the equation.
- WRITING** Describe the number and type of solutions when the value of the discriminant is negative.
- WRITING** Which two methods can you use to solve *any* quadratic equation? Explain when you might prefer to use one method over the other.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–18, solve the equation using the Quadratic Formula. Use a graphing calculator to check your solution(s). (See Examples 1, 2, and 3.)

- | | |
|------------------------|------------------------|
| 5. $x^2 - 4x + 3 = 0$ | 6. $3x^2 + 6x + 3 = 0$ |
| 7. $x^2 + 6x + 15 = 0$ | 8. $6x^2 - 2x + 1 = 0$ |
| 9. $x^2 - 14x = -49$ | 10. $2x^2 + 4x = 30$ |
| 11. $3x^2 + 5 = -2x$ | 12. $-3x = 2x^2 - 4$ |
| 13. $-10x = -25 - x^2$ | 14. $-5x^2 - 6 = -4x$ |
| 15. $-4x^2 + 3x = -5$ | 16. $x^2 + 121 = -22x$ |
| 17. $-z^2 = -12z + 6$ | 18. $-7w + 6 = -4w^2$ |

In Exercises 19–26, find the discriminant of the quadratic equation and describe the number and type of solutions of the equation. (See Example 4.)

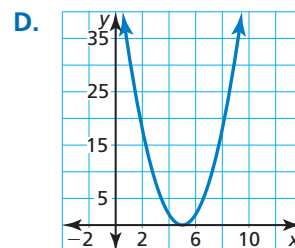
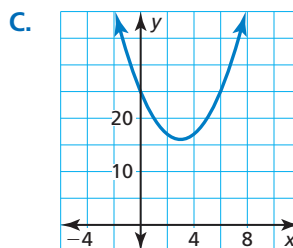
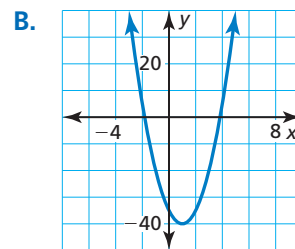
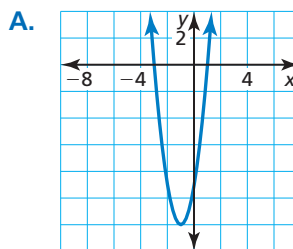
- | | |
|--------------------------|--------------------------|
| 19. $x^2 + 12x + 36 = 0$ | 20. $x^2 - x + 6 = 0$ |
| 21. $4n^2 - 4n - 24 = 0$ | 22. $-x^2 + 2x + 12 = 0$ |
| 23. $4x^2 = 5x - 10$ | 24. $-18p = p^2 + 81$ |
| 25. $24x = -48 - 3x^2$ | 26. $-2x^2 - 6 = x$ |
27. **USING EQUATIONS** What are the complex solutions of the equation $2x^2 - 16x + 50 = 0$?
- | | |
|------------------------|--------------------------|
| (A) $4 + 3i, 4 - 3i$ | (B) $4 + 12i, 4 - 12i$ |
| (C) $16 + 3i, 16 - 3i$ | (D) $16 + 12i, 16 - 12i$ |

28. **USING EQUATIONS** Determine the number and type of solutions to the equation $x^2 + 7x = -11$.


- (A) two real solutions
 (B) one real solution
 (C) two imaginary solutions
 (D) one imaginary solution

ANALYZING EQUATIONS In Exercises 29–32, use the discriminant to match each quadratic equation with the correct graph of the related function. Explain your reasoning.

- | | |
|-------------------------|---------------------------|
| 29. $x^2 - 6x + 25 = 0$ | 30. $2x^2 - 20x + 50 = 0$ |
| 31. $3x^2 + 6x - 9 = 0$ | 32. $5x^2 - 10x - 35 = 0$ |



ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in solving the equation.


33.  $x^2 + 10x + 74 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(74)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{-196}}{2}$$

$$= \frac{-10 \pm 14}{2}$$

$$= -12 \text{ or } 2$$

34.  $x^2 + 6x + 8 = 2$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{4}}{2}$$

$$= \frac{-6 \pm 2}{2}$$

$$= -2 \text{ or } -4$$

OPEN-ENDED In Exercises 35–40, find a possible pair of integer values for a and c so that the quadratic equation has the given solution(s). Then write the equation. (See Example 5.)

35. $ax^2 + 4x + c = 0$; two imaginary solutions
36. $ax^2 + 6x + c = 0$; two real solutions
37. $ax^2 - 8x + c = 0$; two real solutions
38. $ax^2 - 6x + c = 0$; one real solution
39. $ax^2 + 10x = c$; one real solution
40. $-4x + c = -ax^2$; two imaginary solutions

USING STRUCTURE In Exercises 41–46, use the Quadratic Formula to write a quadratic equation that has the given solutions.

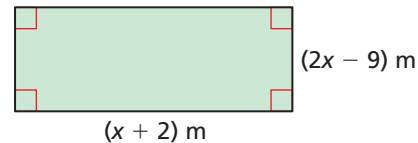
41. $x = \frac{-8 \pm \sqrt{-176}}{-10}$ 42. $x = \frac{15 \pm \sqrt{-215}}{22}$
43. $x = \frac{-4 \pm \sqrt{-124}}{-14}$ 44. $x = \frac{-9 \pm \sqrt{137}}{4}$
45. $x = \frac{-4 \pm 2}{6}$ 46. $x = \frac{2 \pm 4}{-2}$

COMPARING METHODS In Exercises 47–58, solve the quadratic equation using the Quadratic Formula. Then solve the equation using another method. Which method do you prefer? Explain.

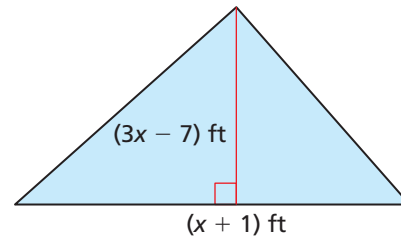
47. $3x^2 - 21 = 3$ 48. $5x^2 + 38 = 3$
49. $2x^2 - 54 = 12x$ 50. $x^2 = 3x + 15$
51. $x^2 - 7x + 12 = 0$ 52. $x^2 + 8x - 13 = 0$
53. $5x^2 - 50x = -135$ 54. $8x^2 + 4x + 5 = 0$
55. $-3 = 4x^2 + 9x$ 56. $-31x + 56 = -x^2$
57. $x^2 = 1 - x$ 58. $9x^2 + 36x + 72 = 0$

MATHEMATICAL CONNECTIONS In Exercises 59 and 60, find the value for x .

59. Area of the rectangle = 24 m^2



60. Area of the triangle = 8 ft^2



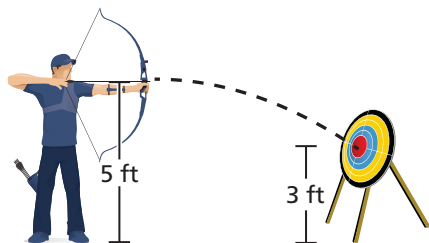
61. **MODELING WITH MATHEMATICS** A lacrosse player throws a ball in the air from an initial height of 7 feet. The ball has an initial vertical velocity of 90 feet per second. Another player catches the ball when it is 3 feet above the ground. How long is the ball in the air? (See Example 6.)



62. **NUMBER SENSE** Suppose the quadratic equation $ax^2 + 5x + c = 0$ has one real solution. Is it possible for a and c to be integers? rational numbers? Explain your reasoning. Then describe the possible values of a and c .

63. MODELING WITH MATHEMATICS In a volleyball game, a player on one team spikes the ball over the net when the ball is 10 feet above the court. The spike drives the ball downward with an initial vertical velocity of 55 feet per second. How much time does the opposing team have to return the ball before it touches the court?

64. MODELING WITH MATHEMATICS An archer is shooting at targets. The height of the arrow is 5 feet above the ground. Due to safety rules, the archer must aim the arrow parallel to the ground.



- How long does it take for the arrow to hit a target that is 3 feet above the ground?
- What method did you use to solve the quadratic equation? Explain.

65. PROBLEM SOLVING A rocketry club is launching model rockets. The launching pad is 30 feet above the ground. Your model rocket has an initial vertical velocity of 105 feet per second. Your friend's model rocket has an initial vertical velocity of 100 feet per second.

- Use a graphing calculator to graph the equations of both model rockets. Compare the paths.
- After how many seconds is your rocket 119 feet above the ground? Explain the reasonableness of your answer(s).

66. PROBLEM SOLVING The number A of tablet computers sold (in millions) can be modeled by the function $A = 4.5t^2 + 43.5t + 17$, where t represents the year after 2010.



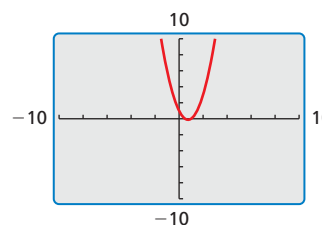
- In what year did the tablet computer sales reach 65 million?
- Find the average rate of change from 2010 to 2012 and interpret the meaning in the context of the situation.
- Do you think this model will be accurate after a new, innovative computer is developed? Explain.

67. MODELING WITH MATHEMATICS A gannet is a bird that feeds on fish by diving into the water. A gannet spots a fish on the surface of the water and dives 100 feet to catch it. The bird plunges toward the water with an initial vertical velocity of -88 feet per second.

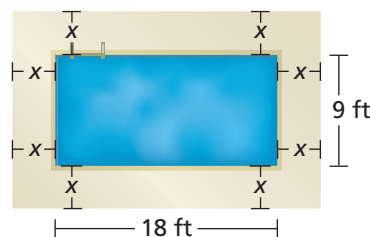


- How much time does the fish have to swim away?
- Another gannet spots the same fish, and it is only 84 feet above the water and has an initial vertical velocity of -70 feet per second. Which bird will reach the fish first? Justify your answer.

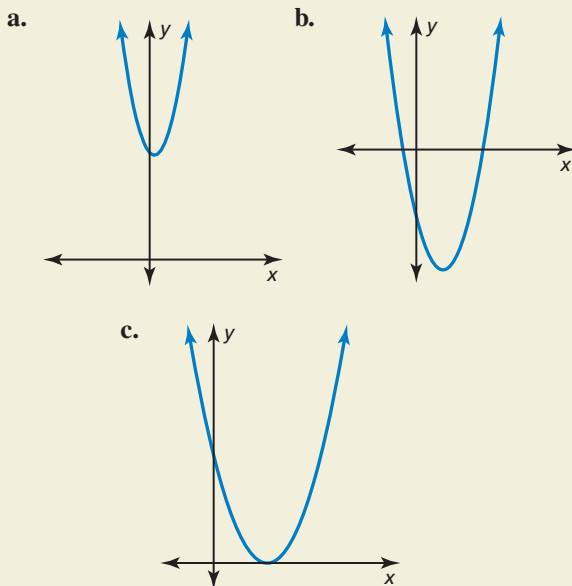
68. USING TOOLS You are asked to find a possible pair of integer values for a and c so that the equation $ax^2 - 3x + c = 0$ has two real solutions. When you solve the inequality for the discriminant, you obtain $ac < 2.25$. So, you choose the values $a = 2$ and $c = 1$. Your graphing calculator displays the graph of your equation in a standard viewing window. Is your solution correct? Explain.



69. PROBLEM SOLVING Your family has a rectangular pool that measures 18 feet by 9 feet. Your family wants to put a deck around the pool but is not sure how wide to make the deck. Determine how wide the deck should be when the total area of the pool and deck is 400 square feet. What is the width of the deck?



- 70. HOW DO YOU SEE IT?** The graph of a quadratic function $y = ax^2 + bx + c$ is shown. Determine whether each discriminant of $ax^2 + bx + c = 0$ is *positive*, *negative*, or *zero*. Then state the number and type of solutions for each graph. Explain your reasoning.



- 71. CRITICAL THINKING** Solve each absolute value equation.

a. $|x^2 - 3x - 14| = 4$ b. $x^2 = |x| + 6$

- 72. MAKING AN ARGUMENT** The class is asked to solve the equation $4x^2 + 14x + 11 = 0$. You decide to solve the equation by completing the square. Your friend decides to use the Quadratic Formula. Whose method is more efficient? Explain your reasoning.

- 73. ABSTRACT REASONING** For a quadratic equation $ax^2 + bx + c = 0$ with two real solutions, show that the mean of the solutions is $-\frac{b}{2a}$. How is this fact related to the symmetry of the graph of $y = ax^2 + bx + c$?

- 74. THOUGHT PROVOKING** Describe a real-life story that could be modeled by $h = -16t^2 + v_0t + h_0$. Write the height model for your story and determine how long your object is in the air.

- 75. REASONING** Show there is no quadratic equation $ax^2 + bx + c = 0$ such that a , b , and c are real numbers and $3i$ and $-2i$ are solutions.

- 76. MODELING WITH MATHEMATICS** The Stratosphere Tower in Las Vegas is 921 feet tall and has a “needle” at its top that extends even higher into the air. A thrill ride called Big Shot catapults riders 160 feet up the needle and then lets them fall back to the launching pad.



- a. The height h (in feet) of a rider on the Big Shot can be modeled by $h = -16t^2 + v_0t + 921$, where t is the elapsed time (in seconds) after launch and v_0 is the initial vertical velocity (in feet per second). Find v_0 using the fact that the maximum value of h is $921 + 160 = 1081$ feet.
- b. A brochure for the Big Shot states that the ride up the needle takes 2 seconds. Compare this time to the time given by the model $h = -16t^2 + v_0t + 921$, where v_0 is the value you found in part (a). Discuss the accuracy of the model.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. (*Skills Review Handbook*)

77. $-x + 2y = 6$
 $x + 4y = 24$

78. $y = 2x - 1$
 $y = x + 1$

79. $3x + y = 4$
 $6x + 2y = -4$

80. $y = -x + 2$
 $-5x + 5y = 10$

Graph the quadratic equation. Label the vertex and axis of symmetry. (*Section 2.2*)

81. $y = -x^2 + 2x + 1$

82. $y = 2x^2 - x + 3$

83. $y = 0.5x^2 + 2x + 5$

84. $y = -3x^2 - 2$

3.5 Solving Nonlinear Systems

Essential Question How can you solve a nonlinear system of equations of equations?

EXPLORATION 1 Solving Nonlinear Systems of Equations

Work with a partner. Match each system with its graph. Explain your reasoning. Then solve each system using the graph.

a. $y = x^2$
 $y = x + 2$

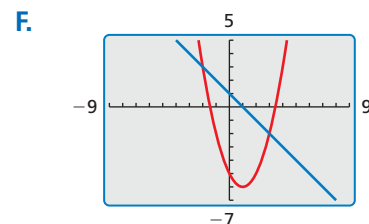
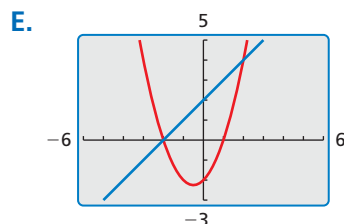
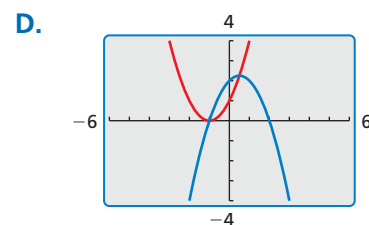
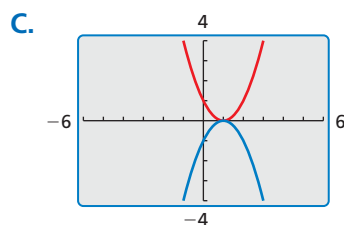
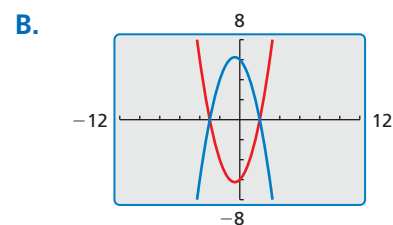
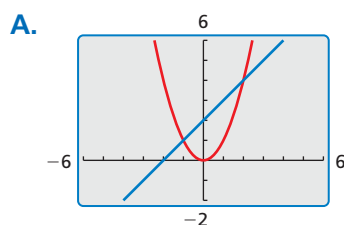
b. $y = x^2 + x - 2$
 $y = x + 2$

c. $y = x^2 - 2x - 5$
 $y = -x + 1$

d. $y = x^2 + x - 6$
 $y = -x^2 - x + 6$

e. $y = x^2 - 2x + 1$
 $y = -x^2 + 2x - 1$

f. $y = x^2 + 2x + 1$
 $y = -x^2 + x + 2$



MAKING SENSE OF PROBLEMS

To be proficient in math, you need to plan a solution pathway rather than simply jumping into a solution attempt.

EXPLORATION 2 Solving Nonlinear Systems of Equations

Work with a partner. Look back at the nonlinear system in Exploration 1(f). Suppose you want a more accurate way to solve the system than using a graphical approach.

- Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the system.
- Show how you could use an *analytical approach*. For instance, you might try solving the system by substitution or elimination.

Communicate Your Answer

- How can you solve a nonlinear system of equations?
- Would you prefer to use a graphical, numerical, or analytical approach to solve the given nonlinear system of equations? Explain your reasoning.

$$y = x^2 + 2x - 3$$

$$y = -x^2 - 2x + 4$$

3.5 Lesson

Core Vocabulary

system of nonlinear equations,
p. 132

Previous

system of linear equations
circle

What You Will Learn

- ▶ Solve systems of nonlinear equations.
- ▶ Solve quadratic equations by graphing.

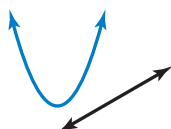
Systems of Nonlinear Equations

Previously, you solved systems of *linear* equations by graphing, substitution, and elimination. You can also use these methods to solve a system of *nonlinear* equations. In a **system of nonlinear equations**, at least one of the equations is nonlinear. For instance, the nonlinear system shown has a quadratic equation and a linear equation.

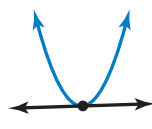
$$y = x^2 + 2x - 4 \quad \text{Equation 1 is nonlinear.}$$

$$y = 2x + 5 \quad \text{Equation 2 is linear.}$$

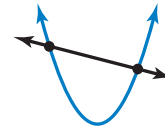
When the graphs of the equations in a system are a line and a parabola, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solution

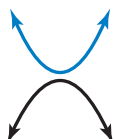


One solution

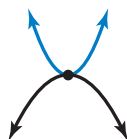


Two solutions

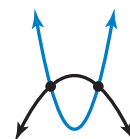
When the graphs of the equations in a system are a parabola that opens up and a parabola that opens down, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solution



One solution



Two solutions

EXAMPLE 1

Solving a Nonlinear System by Graphing

Solve the system by graphing.

$$y = x^2 - 2x - 1 \quad \text{Equation 1}$$

$$y = -2x - 1 \quad \text{Equation 2}$$

SOLUTION

Graph each equation. Then estimate the point of intersection. The parabola and the line appear to intersect at the point $(0, -1)$. Check the point by substituting the coordinates into each of the original equations.

Equation 1

$$y = x^2 - 2x - 1$$

$$-1 \stackrel{?}{=} (0)^2 - 2(0) - 1$$

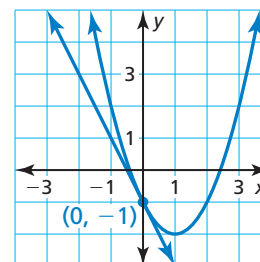
$$-1 = -1 \quad \checkmark$$

Equation 2

$$y = -2x - 1$$

$$-1 \stackrel{?}{=} -2(0) - 1$$

$$-1 = -1 \quad \checkmark$$



- ▶ The solution is $(0, -1)$.

EXAMPLE 2 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$$x^2 + x - y = -1 \quad \text{Equation 1}$$
$$x + y = 4 \quad \text{Equation 2}$$

SOLUTION

Begin by solving for y in Equation 2.

$$y = -x + 4 \quad \text{Solve for } y \text{ in Equation 2.}$$

Next, substitute $-x + 4$ for y in Equation 1 and solve for x .

$$x^2 + x - y = -1 \quad \text{Write Equation 1.}$$

$$x^2 + x - (-x + 4) = -1 \quad \text{Substitute } -x + 4 \text{ for } y.$$

$$x^2 + 2x - 4 = -1 \quad \text{Simplify.}$$

$$x^2 + 2x - 3 = 0 \quad \text{Write in standard form.}$$

$$(x + 3)(x - 1) = 0 \quad \text{Factor.}$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{Zero-Product Property}$$

$$x = -3 \quad \text{or} \quad x = 1 \quad \text{Solve for } x.$$

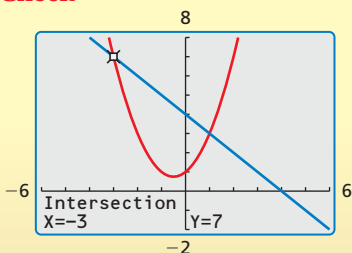
To solve for y , substitute $x = -3$ and $x = 1$ into the equation $y = -x + 4$.

$$y = -x + 4 = -(-3) + 4 = 7 \quad \text{Substitute } -3 \text{ for } x.$$

$$y = -x + 4 = -1 + 4 = 3 \quad \text{Substitute } 1 \text{ for } x.$$

► The solutions are $(-3, 7)$ and $(1, 3)$. Check the solutions by graphing the system.

Check



EXAMPLE 3 Solving a Nonlinear System by Elimination

Solve the system by elimination.

$$2x^2 - 5x - y = -2 \quad \text{Equation 1}$$
$$x^2 + 2x + y = 0 \quad \text{Equation 2}$$

SOLUTION

Add the equations to eliminate the y -term and obtain a quadratic equation in x .

$$2x^2 - 5x - y = -2$$

$$x^2 + 2x + y = 0$$

$$3x^2 - 3x = -2$$

$$3x^2 - 3x + 2 = 0$$

$$x = \frac{3 \pm \sqrt{-15}}{6}$$

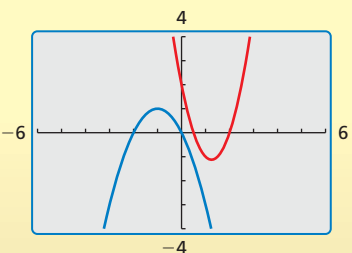
Add the equations.

Write in standard form.

Use the Quadratic Formula.

► Because the discriminant is negative, the equation $3x^2 - 3x + 2 = 0$ has no real solution. So, the original system has no real solution. You can check this by graphing the system and seeing that the graphs do not appear to intersect.

Check



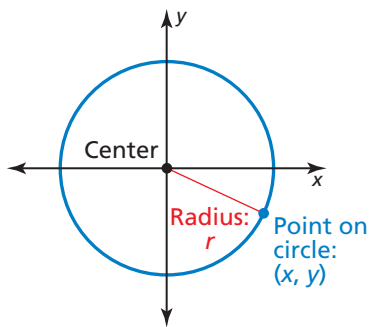
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Solve the system using any method. Explain your choice of method.

1. $y = -x^2 + 4$
 $y = -4x + 8$

2. $x^2 + 3x + y = 0$
 $2x + y = 5$

3. $2x^2 + 4x - y = -2$
 $x^2 + y = 2$

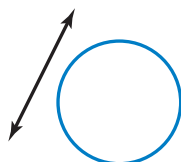


Some nonlinear systems have equations of the form

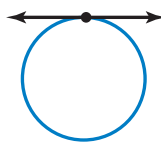
$$x^2 + y^2 = r^2.$$

This equation is the standard form of a circle with center $(0, 0)$ and radius r .

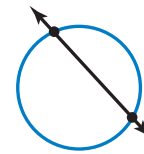
When the graphs of the equations in a system are a line and a circle, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solution



One solution



Two solutions

EXAMPLE 4 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$$x^2 + y^2 = 10 \quad \text{Equation 1}$$

$$y = -3x + 10 \quad \text{Equation 2}$$

SOLUTION

Substitute $-3x + 10$ for y in Equation 1 and solve for x .

$$x^2 + y^2 = 10$$

Write Equation 1.

$$x^2 + (-3x + 10)^2 = 10$$

Substitute $-3x + 10$ for y .

$$x^2 + 9x^2 - 60x + 100 = 10$$

Expand the power.

$$10x^2 - 60x + 90 = 10$$

Write in standard form.

$$x^2 - 6x + 9 = 0$$

Divide each side by 10.

$$(x - 3)^2 = 0$$

Perfect Square Trinomial Pattern

$$x = 3$$

Zero-Product Property

COMMON ERROR

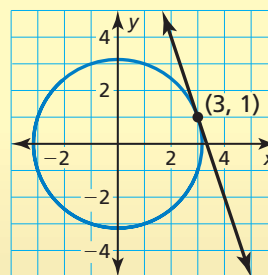
You can also substitute $x = 3$ in Equation 1 to find y . This yields two *apparent* solutions, $(3, 1)$ and $(3, -1)$. However, $(3, -1)$ is *not* a solution because it does not satisfy Equation 2. You can also see $(3, -1)$ is not a solution from the graph.

To find the y -coordinate of the solution, substitute $x = 3$ in Equation 2.

$$y = -3(3) + 10 = 1$$

- The solution is $(3, 1)$. Check the solution by graphing the system. You can see that the line and the circle intersect only at the point $(3, 1)$.

Check



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Solve the system.

4. $x^2 + y^2 = 16$

$$y = -x + 4$$

5. $x^2 + y^2 = 4$

$$y = x + 4$$

6. $x^2 + y^2 = 1$

$$y = \frac{1}{2}x + \frac{1}{2}$$

Solving Equations by Graphing

You can solve an equation by rewriting it as a system of equations and then solving the system by graphing.

Core Concept

Solving Equations by Graphing

Step 1 To solve the equation $f(x) = g(x)$, write a system of two equations, $y = f(x)$ and $y = g(x)$.

Step 2 Graph the system of equations $y = f(x)$ and $y = g(x)$. The x -value of each solution of the system is a solution of the equation $f(x) = g(x)$.

ANOTHER WAY

In Example 5(a), you can also find the solutions by writing the given equation as $4x^2 + 3x - 2 = 0$ and solving this equation using the Quadratic Formula.

EXAMPLE 5 Solving Quadratic Equations by Graphing

Solve (a) $3x^2 + 5x - 1 = -x^2 + 2x + 1$ and (b) $-(x - 1.5)^2 + 2.25 = 2x(x + 1.5)$ by graphing.

SOLUTION

a. Step 1 Write a system of equations using each side of the original equation.

Equation

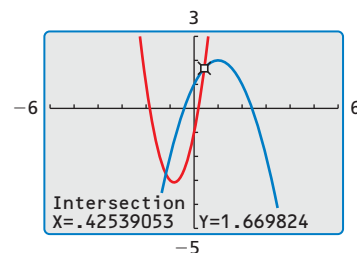
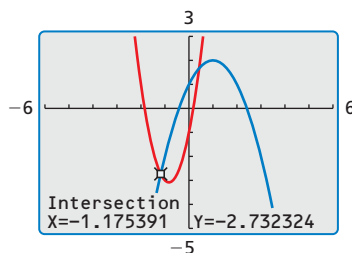
$$3x^2 + 5x - 1 = -x^2 + 2x + 1$$

System

$$y = 3x^2 + 5x - 1$$

$$y = -x^2 + 2x + 1$$

Step 2 Use a graphing calculator to graph the system. Then use the *intersect* feature to find the x -value of each solution of the system.



The graphs intersect when $x \approx -1.18$ and $x \approx 0.43$.

► The solutions of the equation are $x \approx -1.18$ and $x \approx 0.43$.

b. Step 1 Write a system of equations using each side of the original equation.

Equation

$$-(x - 1.5)^2 + 2.25 = 2x(x + 1.5)$$

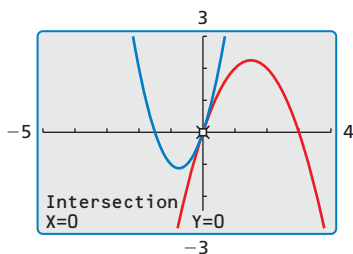
System

$$y = -(x - 1.5)^2 + 2.25$$

$$y = 2x(x + 1.5)$$

Step 2 Use a graphing calculator to graph the system, as shown at the left. Then use the *intersect* feature to find the x -value of each solution of the system. The graphs intersect when $x = 0$.

► The solution of the equation is $x = 0$.



Monitoring Progress



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Solve the equation by graphing.

7. $x^2 - 6x + 15 = -(x - 3)^2 + 6$

8. $(x + 4)(x - 1) = -x^2 + 3x + 4$

Vocabulary and Core Concept Check

- WRITING** Describe the possible solutions of a system consisting of two quadratic equations.
- WHICH ONE DOESN'T BELONG?** Which system does *not* belong with the other three? Explain your reasoning.

$$\begin{aligned} y &= 3x + 4 \\ y &= x^2 + 1 \end{aligned}$$

$$\begin{aligned} y &= 2x - 1 \\ y &= -3x + 6 \end{aligned}$$

$$\begin{aligned} y &= 3x^2 + 4x + 1 \\ y &= -5x^2 - 3x + 1 \end{aligned}$$

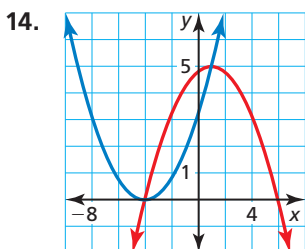
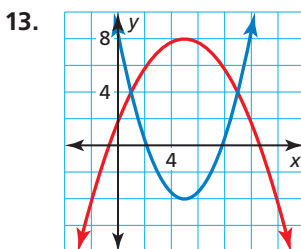
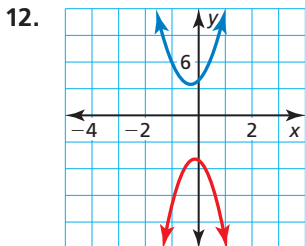
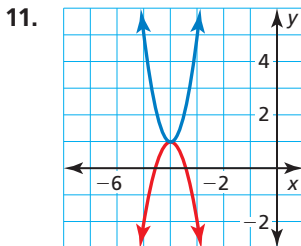
$$\begin{aligned} x^2 + y^2 &= 4 \\ y &= -x + 1 \end{aligned}$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the system by graphing. Check your solution(s). (See Example 1.)

- $y = x + 2$
 $y = 0.5(x + 2)^2$
- $y = (x - 3)^2 + 5$
 $y = 5$
- $y = \frac{1}{3}x + 2$
 $y = -3x^2 - 5x - 4$
- $y = -3x^2 - 30x - 71$
 $y = -3x - 17$
- $y = x^2 + 8x + 18$
 $y = -2x^2 - 16x - 30$
- $y = -2x^2 - 9$
 $y = -4x - 1$
- $y = (x - 2)^2$
 $y = -x^2 + 4x - 2$
- $y = \frac{1}{2}(x + 2)^2$
 $y = -\frac{1}{2}x^2 + 2$

In Exercises 11–14, solve the system of nonlinear equations using the graph.



In Exercises 15–24, solve the system by substitution. (See Examples 2 and 4.)

- $y = x + 5$
 $y = x^2 - x + 2$
- $x^2 + y^2 = 49$
 $y = 7 - x$
- $x^2 + y^2 = 64$
 $y = -8$
- $x = 3$
 $-3x^2 + 4x - y = 8$
- $2x^2 + 4x - y = -3$
 $-2x + y = -4$
- $2x - 3 = y + 5x^2$
 $y = -3x - 3$
- $y = x^2 - 1$
 $-7 = -x^2 - y$
- $y + 16x - 22 = 4x^2$
 $4x^2 - 24x + 26 + y = 0$
- $x^2 + y^2 = 7$
 $x + 3y = 21$
- $x^2 + y^2 = 5$
 $-x + y = -1$

25. **USING EQUATIONS** Which ordered pairs are solutions of the nonlinear system?

$$y = \frac{1}{2}x^2 - 5x + \frac{21}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2}$$

- (A) (1, 6) (B) (3, 0)
(C) (8, 2.5) (D) (7, 0)

26. **USING EQUATIONS** How many solutions does the system have? Explain your reasoning.

$$y = 7x^2 - 11x + 9$$

$$y = -7x^2 + 5x - 3$$

- (A) 0 (B) 1
(C) 2 (D) 4

In Exercises 27–34, solve the system by elimination.
(See Example 3.)

27. $2x^2 - 3x - y = -5$ 28. $-3x^2 + 2x - 5 = y$
 $-x + y = 5$ $-x + 2 = -y$

29. $-3x^2 + y = -18x + 29$
 $-3x^2 - y = 18x - 25$

30. $y = -x^2 - 6x - 10$
 $y = 3x^2 + 18x + 22$

31. $y + 2x = -14$ 32. $y = x^2 + 4x + 7$
 $-x^2 - y - 6x = 11$ $-y = 4x + 7$

33. $y = -3x^2 - 30x - 76$
 $y = 2x^2 + 20x + 44$

34. $-10x^2 + y = -80x + 155$
 $5x^2 + y = 40x - 85$

35. **ERROR ANALYSIS** Describe and correct the error in using elimination to solve a system.

✗

$$y = -2x^2 + 32x - 126$$

$$-y = 2x - 14$$

$$0 = 18x - 126$$

$$126 = 18x$$

$$x = 7$$

36. **NUMBER SENSE** The table shows the inputs and outputs of two quadratic equations. Identify the solution(s) of the system. Explain your reasoning.

x	y_1	y_2
-3	29	-11
-1	9	9
1	-3	21
3	-7	25
7	9	9
11	57	-39

In Exercises 37–42, solve the system using any method.
Explain your choice of method.

37. $y = x^2 - 1$ 38. $y = -4x^2 - 16x - 13$
 $-y = 2x^2 + 1$ $-3x^2 + y + 12x = 17$

39. $-2x + 10 + y = \frac{1}{3}x^2$ 40. $y = 0.5x^2 - 10$
 $y = 10$ $y = -x^2 + 14$

41. $y = -3(x - 4)^2 + 6$
 $(x - 4)^2 + 2 - y = 0$

42. $-x^2 + y^2 = 100$
 $y = -x + 14$

USING TOOLS In Exercises 43–48, solve the equation by graphing. (See Example 5.)

43. $x^2 + 2x = -\frac{1}{2}x^2 + 2x$

44. $2x^2 - 12x - 16 = -6x^2 + 60x - 144$

45. $(x + 2)(x - 2) = -x^2 + 6x - 7$

46. $-2x^2 - 16x - 25 = 6x^2 + 48x + 95$

47. $(x - 2)^2 - 3 = (x + 3)(-x + 9) - 38$

48. $(-x + 4)(x + 8) - 42 = (x + 3)(x + 1) - 1$

49. **REASONING** A nonlinear system contains the equations of a constant function and a quadratic function. The system has one solution. Describe the relationship between the graphs.

50. **PROBLEM SOLVING** The range (in miles) of a broadcast signal from a radio tower is bounded by a circle given by the equation

$$x^2 + y^2 = 1620.$$

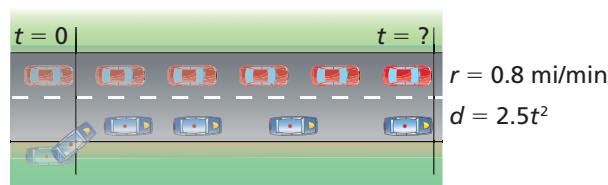
A straight highway can be modeled by the equation

$$y = -\frac{1}{3}x + 30.$$

For what lengths of the highway are cars able to receive the broadcast signal?



51. **PROBLEM SOLVING** A car passes a parked police car and continues at a constant speed r . The police car begins accelerating at a constant rate when it is passed. The diagram indicates the distance d (in miles) the police car travels as a function of time t (in minutes) after being passed. Write and solve a system of equations to find how long it takes the police car to catch up to the other car.



52. THOUGHT PROVOKING Write a nonlinear system that has two different solutions with the same y -coordinate. Sketch a graph of your system. Then solve the system.

53. OPEN-ENDED Find three values for m so the system has no solution, one solution, and two solutions. Justify your answer using a graph.

$$3y = -x^2 + 8x - 7$$

$$y = mx + 3$$

54. MAKING AN ARGUMENT You and a friend solve the system shown and determine that $x = 3$ and $x = -3$. You use Equation 1 to obtain the solutions $(3, 3)$, $(3, -3)$, $(-3, 3)$, and $(-3, -3)$. Your friend uses Equation 2 to obtain the solutions $(3, 3)$ and $(-3, -3)$. Who is correct? Explain your reasoning.

$$x^2 + y^2 = 18 \quad \text{Equation 1}$$

$$x - y = 0 \quad \text{Equation 2}$$

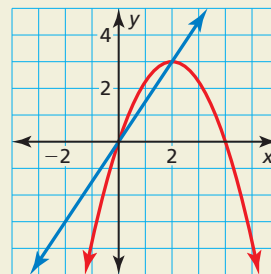
55. COMPARING METHODS Describe two different ways you could solve the quadratic equation. Which way do you prefer? Explain your reasoning.

$$-2x^2 + 12x - 17 = 2x^2 - 16x + 31$$

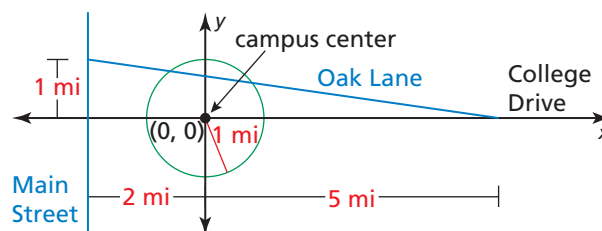
56. ANALYZING RELATIONSHIPS Suppose the graph of a line that passes through the origin intersects the graph of a circle with its center at the origin. When you know one of the points of intersection, explain how you can find the other point of intersection without performing any calculations.

57. WRITING Describe the possible solutions of a system that contains (a) one quadratic equation and one equation of a circle, and (b) two equations of circles. Sketch graphs to justify your answers.

58. HOW DO YOU SEE IT? The graph of a nonlinear system is shown. Estimate the solution(s). Then describe the transformation of the graph of the linear function that results in a system with no solution.



59. MODELING WITH MATHEMATICS To be eligible for a parking pass on a college campus, a student must live at least 1 mile from the campus center.



- Write equations that represent the circle and Oak Lane.
- Solve the system that consists of the equations in part (a).
- For what length of Oak Lane are students *not* eligible for a parking pass?

60. CRITICAL THINKING Solve the system of three equations shown.

$$x^2 + y^2 = 4$$

$$2y = x^2 - 2x + 4$$

$$y = -x + 2$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution on a number line. (*Skills Review Handbook*)

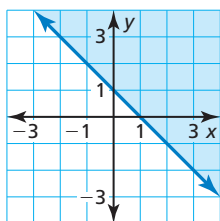
61. $4x - 4 > 8$

62. $-x + 7 \leq 4 - 2x$

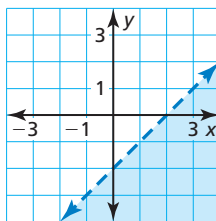
63. $-3(x - 4) \geq 24$

Write an inequality that represents the graph. (*Skills Review Handbook*)

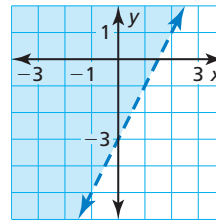
64.



65.



66.



3.6 Quadratic Inequalities

Essential Question How can you solve a quadratic inequality?

EXPLORATION 1 Solving a Quadratic Inequality

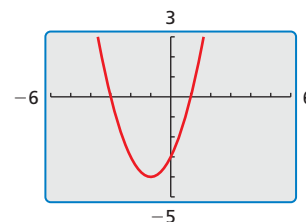
Work with a partner. The graphing calculator screen shows the graph of

$$f(x) = x^2 + 2x - 3.$$

Explain how you can use the graph to solve the inequality

$$x^2 + 2x - 3 \leq 0.$$

Then solve the inequality.



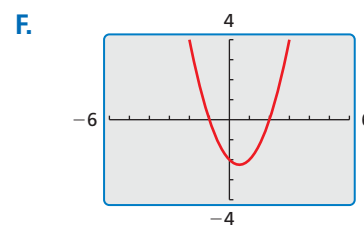
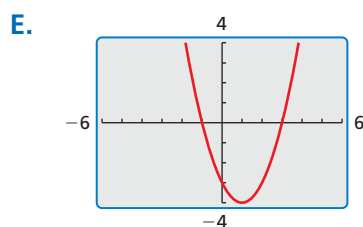
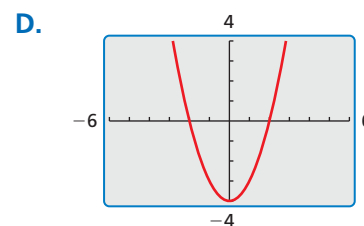
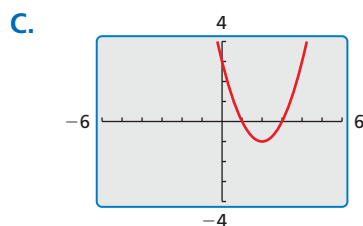
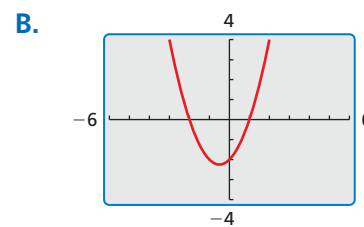
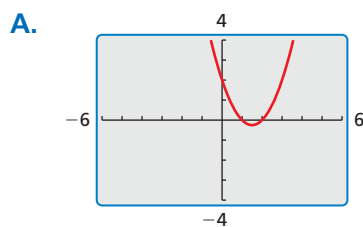
USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technological tools to explore your understanding of concepts.

EXPLORATION 2 Solving Quadratic Inequalities

Work with a partner. Match each inequality with the graph of its related quadratic function. Then use the graph to solve the inequality.

- a. $x^2 - 3x + 2 > 0$ b. $x^2 - 4x + 3 \leq 0$ c. $x^2 - 2x - 3 < 0$
 d. $x^2 + x - 2 \geq 0$ e. $x^2 - x - 2 < 0$ f. $x^2 - 4 > 0$



Communicate Your Answer

- How can you solve a quadratic inequality?
- Explain how you can use the graph in Exploration 1 to solve each inequality. Then solve each inequality.
 - $x^2 + 2x - 3 > 0$
 - $x^2 + 2x - 3 < 0$
 - $x^2 + 2x - 3 \geq 0$

3.6 Lesson

Core Vocabulary

quadratic inequality in two variables, p. 140
quadratic inequality in one variable, p. 142

Previous

linear inequality in two variables

What You Will Learn

- ▶ Graph quadratic inequalities in two variables.
- ▶ Solve quadratic inequalities in one variable.

Graphing Quadratic Inequalities in Two Variables

A **quadratic inequality in two variables** can be written in one of the following forms, where a , b , and c are real numbers and $a \neq 0$.

$$y < ax^2 + bx + c \qquad y > ax^2 + bx + c$$

$$y \leq ax^2 + bx + c \qquad y \geq ax^2 + bx + c$$

The graph of any such inequality consists of all solutions (x, y) of the inequality.

Previously, you graphed linear inequalities in two variables. You can use a similar procedure to graph quadratic inequalities in two variables.

Core Concept

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the forms above, follow these steps.

- Step 1** Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with \leq or \geq .
- Step 2** Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.
- Step 3** Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

EXAMPLE 1 Graphing a Quadratic Inequality in Two Variables

Graph $y < -x^2 - 2x - 1$.

SOLUTION

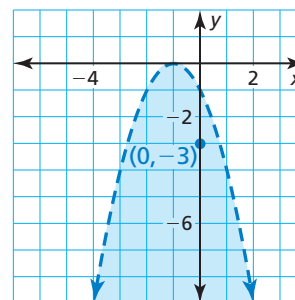
Step 1 Graph $y = -x^2 - 2x - 1$. Because the inequality symbol is $<$, make the parabola dashed.

Step 2 Test a point inside the parabola, such as $(0, -3)$.

$$\begin{aligned} y &< -x^2 - 2x - 1 \\ -3 &\stackrel{?}{<} -0^2 - 2(0) - 1 \\ -3 &< -1 \quad \checkmark \end{aligned}$$

So, $(0, -3)$ is a solution of the inequality.

Step 3 Shade the region inside the parabola.



LOOKING FOR STRUCTURE

Notice that testing a point is less complicated when the x -value is 0 (the point is on the y -axis).

EXAMPLE 2 Using a Quadratic Inequality in Real Life

A manila rope used for rappelling down a cliff can safely support a weight W (in pounds) provided

$$W \leq 1480d^2$$

where d is the diameter (in inches) of the rope. Graph the inequality and interpret the solution.

SOLUTION

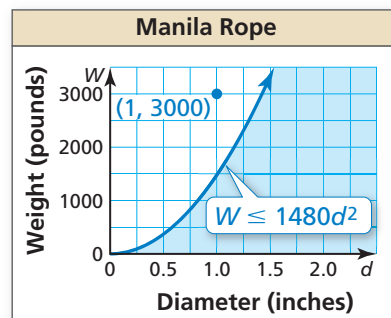
Graph $W = 1480d^2$ for nonnegative values of d . Because the inequality symbol is \leq , make the parabola solid. Test a point inside the parabola, such as $(1, 3000)$.

$$W \leq 1480d^2$$

$$3000 \stackrel{?}{\leq} 1480(1)^2$$

$$3000 \not\leq 1480$$

- Because $(1, 3000)$ is not a solution, shade the region outside the parabola. The shaded region represents weights that can be supported by ropes with various diameters.



Graphing a *system* of quadratic inequalities is similar to graphing a system of linear inequalities. First graph each inequality in the system. Then identify the region in the coordinate plane common to all of the graphs. This region is called the *graph of the system*.

EXAMPLE 3 Graphing a System of Quadratic Inequalities

Graph the system of quadratic inequalities.

$$y < -x^2 + 3 \quad \text{Inequality 1}$$

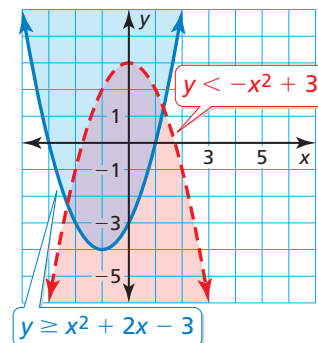
$$y \geq x^2 + 2x - 3 \quad \text{Inequality 2}$$

SOLUTION

Step 1 Graph $y < -x^2 + 3$. The graph is the red region inside (but not including) the parabola $y = -x^2 + 3$.

Step 2 Graph $y \geq x^2 + 2x - 3$. The graph is the blue region inside and including the parabola $y = x^2 + 2x - 3$.

Step 3 Identify the purple region where the two graphs overlap. This region is the graph of the system.



Check

Check that a point in the solution region, such as $(0, 0)$, is a solution of the system.

$$y < -x^2 + 3$$

$$0 \stackrel{?}{<} -0^2 + 3$$

$$0 < 3 \quad \checkmark$$

$$y \geq x^2 + 2x - 3$$

$$0 \stackrel{?}{\geq} 0^2 + 2(0) - 3$$

$$0 \geq -3 \quad \checkmark$$

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Graph the inequality.

1. $y \geq x^2 + 2x - 8$

2. $y \leq 2x^2 - x - 1$

3. $y > -x^2 + 2x + 4$

4. Graph the system of inequalities consisting of $y \leq -x^2$ and $y > x^2 - 3$.

Solving Quadratic Inequalities in One Variable

A **quadratic inequality in one variable** can be written in one of the following forms, where a , b , and c are real numbers and $a \neq 0$.

$$ax^2 + bx + c < 0 \quad ax^2 + bx + c > 0 \quad ax^2 + bx + c \leq 0 \quad ax^2 + bx + c \geq 0$$

You can solve quadratic inequalities using algebraic methods or graphs.

EXAMPLE 4 Solving a Quadratic Inequality Algebraically

Solve $x^2 - 3x - 4 < 0$ algebraically.

SOLUTION

First, write and solve the equation obtained by replacing $<$ with $=$.

$$x^2 - 3x - 4 = 0$$

Write the related equation.

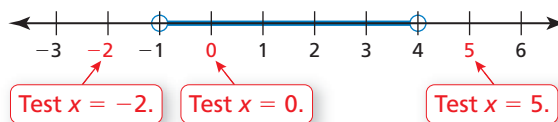
$$(x - 4)(x + 1) = 0$$

Factor.

$$x = 4 \quad \text{or} \quad x = -1$$

Zero-Product Property

The numbers -1 and 4 are the *critical values* of the original inequality. Plot -1 and 4 on a number line, using open dots because the values do not satisfy the inequality. The critical x -values partition the number line into three intervals. Test an x -value in each interval to determine whether it satisfies the inequality.



$$(-2)^2 - 3(-2) - 4 = 6 \not< 0 \quad 0^2 - 3(0) - 4 = -4 < 0 \quad 5^2 - 3(5) - 4 = 6 \not< 0$$

► So, the solution is $-1 < x < 4$.

Another way to solve $ax^2 + bx + c < 0$ is to first graph the related function $y = ax^2 + bx + c$. Then, because the inequality symbol is $<$, identify the x -values for which the graph lies *below* the x -axis. You can use a similar procedure to solve quadratic inequalities that involve \leq , $>$, or \geq .

EXAMPLE 5 Solving a Quadratic Inequality by Graphing

Solve $3x^2 - x - 5 \geq 0$ by graphing.

SOLUTION

The solution consists of the x -values for which the graph of $y = 3x^2 - x - 5$ lies on or above the x -axis. Find the x -intercepts of the graph by letting $y = 0$ and using the Quadratic Formula to solve $0 = 3x^2 - x - 5$ for x .

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)}$$

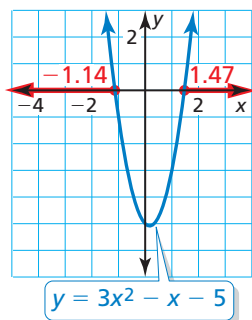
$$a = 3, b = -1, c = -5$$

$$x = \frac{1 \pm \sqrt{61}}{6}$$

Simplify.

The solutions are $x \approx -1.14$ and $x \approx 1.47$. Sketch a parabola that opens up and has -1.14 and 1.47 as x -intercepts. The graph lies on or above the x -axis to the left of (and including) $x = -1.14$ and to the right of (and including) $x = 1.47$.

► The solution of the inequality is approximately $x \leq -1.14$ or $x \geq 1.47$.



EXAMPLE 6 Modeling with Mathematics

A rectangular parking lot must have a perimeter of 440 feet and an area of at least 8000 square feet. Describe the possible lengths of the parking lot.

SOLUTION

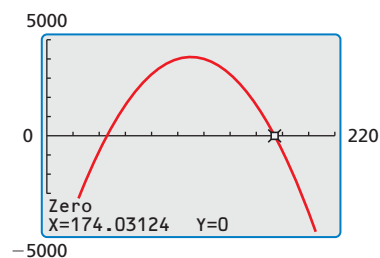
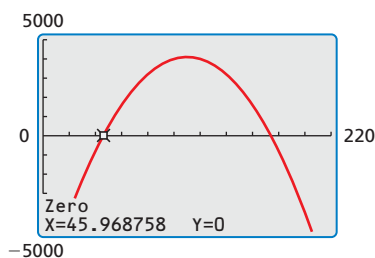
- 1. Understand the Problem** You are given the perimeter and the minimum area of a parking lot. You are asked to determine the possible lengths of the parking lot.
- 2. Make a Plan** Use the perimeter and area formulas to write a quadratic inequality describing the possible lengths of the parking lot. Then solve the inequality.
- 3. Solve the Problem** Let ℓ represent the length (in feet) and let w represent the width (in feet) of the parking lot.

$$\begin{array}{ll} \text{Perimeter} = 440 & \text{Area} \geq 8000 \\ 2\ell + 2w = 440 & \ell w \geq 8000 \end{array}$$

Solve the perimeter equation for w to obtain $w = 220 - \ell$. Substitute this into the area inequality to obtain a quadratic inequality in one variable.

$$\begin{array}{ll} \ell w \geq 8000 & \text{Write the area inequality.} \\ \ell(220 - \ell) \geq 8000 & \text{Substitute } 220 - \ell \text{ for } w. \\ 220\ell - \ell^2 \geq 8000 & \text{Distributive Property} \\ -\ell^2 + 220\ell - 8000 \geq 0 & \text{Write in standard form.} \end{array}$$

Use a graphing calculator to find the ℓ -intercepts of $y = -\ell^2 + 220\ell - 8000$.



The ℓ -intercepts are $\ell \approx 45.97$ and $\ell \approx 174.03$. The solution consists of the ℓ -values for which the graph lies on or above the ℓ -axis. The graph lies on or above the ℓ -axis when $45.97 \leq \ell \leq 174.03$.

► So, the approximate length of the parking lot is at least 46 feet and at most 174 feet.

- 4. Look Back** Choose a length in the solution region, such as $\ell = 100$, and find the width. Then check that the dimensions satisfy the original area inequality.

$$\begin{array}{ll} 2\ell + 2w = 440 & \ell w \geq 8000 \\ 2(100) + 2w = 440 & 100(120) \stackrel{?}{\geq} 8000 \\ w = 120 & 12,000 \geq 8000 \quad \checkmark \end{array}$$

ANOTHER WAY

You can graph each side of $220\ell - \ell^2 = 8000$ and use the intersection points to determine when $220\ell - \ell^2$ is greater than or equal to 8000.

USING TECHNOLOGY

Variables displayed when using technology may not match the variables used in applications. In the graphs shown, the length ℓ corresponds to the independent variable x .

Monitoring Progress Help in English and Spanish at BigIdeasMath.com

Solve the inequality.

$$5. 2x^2 + 3x \leq 2 \qquad 6. -3x^2 - 4x + 1 < 0 \qquad 7. 2x^2 + 2 > -5x$$

- 8. WHAT IF?** In Example 6, the area must be at least 8500 square feet. Describe the possible lengths of the parking lot.

3.6 Exercises

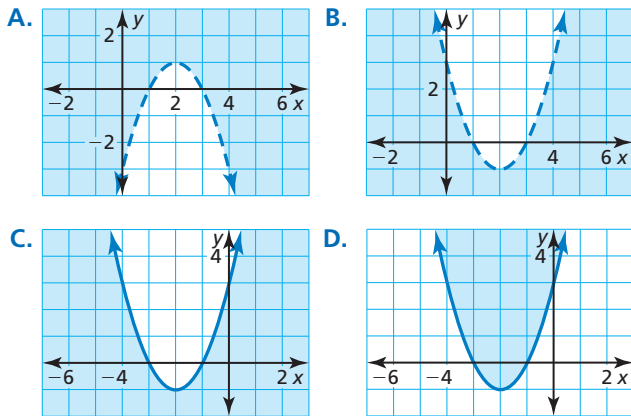
Vocabulary and Core Concept Check

- WRITING** Compare the graph of a quadratic inequality in one variable to the graph of a quadratic inequality in two variables.
- WRITING** Explain how to solve $x^2 + 6x - 8 < 0$ using algebraic methods and using graphs.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the inequality with its graph. Explain your reasoning.

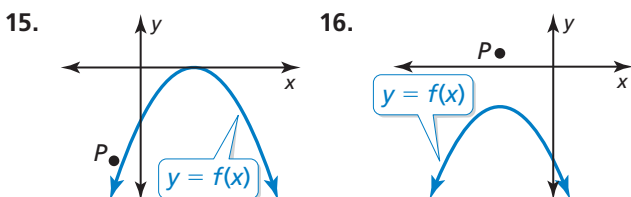
- | | |
|--------------------------|--------------------------|
| 3. $y \leq x^2 + 4x + 3$ | 4. $y > -x^2 + 4x - 3$ |
| 5. $y < x^2 - 4x + 3$ | 6. $y \geq x^2 + 4x + 3$ |



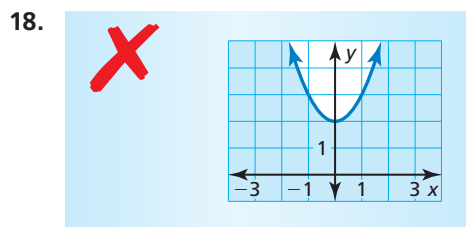
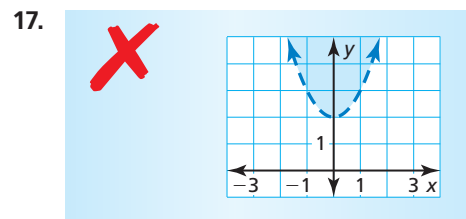
In Exercises 7–14, graph the inequality. (See Example 1.)

- | | |
|--------------------------|---|
| 7. $y < -x^2$ | 8. $y \geq 4x^2$ |
| 9. $y > x^2 - 9$ | 10. $y < x^2 + 5$ |
| 11. $y \leq x^2 + 5x$ | 12. $y \geq -2x^2 + 9x - 4$ |
| 13. $y > 2(x + 3)^2 - 1$ | 14. $y \leq \left(x - \frac{1}{2}\right)^2 + \frac{5}{2}$ |

ANALYZING RELATIONSHIPS In Exercises 15 and 16, use the graph to write an inequality in terms of $f(x)$ so point P is a solution.



ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in graphing $y \geq x^2 + 2$.



- MODELING WITH MATHEMATICS** A hardwood shelf in a wooden bookcase can safely support a weight W (in pounds) provided $W \leq 115x^2$, where x is the thickness (in inches) of the shelf. Graph the inequality and interpret the solution. (See Example 2.)
- MODELING WITH MATHEMATICS** A wire rope can safely support a weight W (in pounds) provided $W \leq 8000d^2$, where d is the diameter (in inches) of the rope. Graph the inequality and interpret the solution.

In Exercises 21–26, graph the system of quadratic inequalities. (See Example 3.)

- | | |
|---|--|
| 21. $y \geq 2x^2$
$y < -x^2 + 1$ | 22. $y > -5x^2$
$y > 3x^2 - 2$ |
| 23. $y \leq -x^2 + 4x - 4$
$y < x^2 + 2x - 8$ | 24. $y \geq x^2 - 4$
$y \leq -2x^2 + 7x + 4$ |
| 25. $y \geq 2x^2 + x - 5$
$y < -x^2 + 5x + 10$ | 26. $y \geq x^2 - 3x - 6$
$y \geq x^2 + 7x + 6$ |

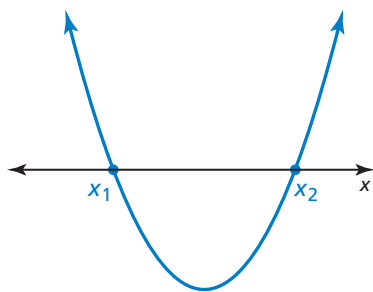
In Exercises 27–34, solve the inequality algebraically. (See Example 4.)

27. $4x^2 < 25$ 28. $x^2 + 10x + 9 < 0$
 29. $x^2 - 11x \geq -28$ 30. $3x^2 - 13x > -10$
 31. $2x^2 - 5x - 3 \leq 0$ 32. $4x^2 + 8x - 21 \geq 0$
 33. $\frac{1}{2}x^2 - x > 4$ 34. $-\frac{1}{2}x^2 + 4x \leq 1$

In Exercises 35–42, solve the inequality by graphing. (See Example 5.)

35. $x^2 - 3x + 1 < 0$ 36. $x^2 - 4x + 2 > 0$
 37. $x^2 + 8x > -7$ 38. $x^2 + 6x < -3$
 39. $3x^2 - 8 \leq -2x$ 40. $3x^2 + 5x - 3 < 1$
 41. $\frac{1}{3}x^2 + 2x \geq 2$ 42. $\frac{3}{4}x^2 + 4x \geq 3$

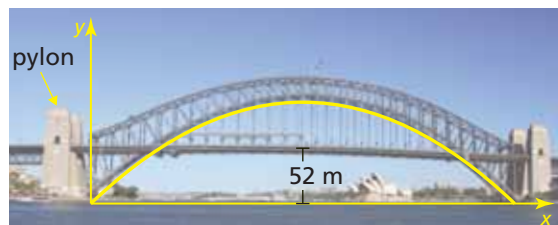
43. **DRAWING CONCLUSIONS** Consider the graph of the function $f(x) = ax^2 + bx + c$.



- a. What are the solutions of $ax^2 + bx + c < 0$?
 b. What are the solutions of $ax^2 + bx + c > 0$?
 c. The graph of g represents a reflection in the x -axis of the graph of f . For which values of x is $g(x)$ positive?
44. **MODELING WITH MATHEMATICS** A rectangular fountain display has a perimeter of 400 feet and an area of at least 9100 feet. Describe the possible widths of the fountain. (See Example 6.)



45. **MODELING WITH MATHEMATICS** The arch of the Sydney Harbor Bridge in Sydney, Australia, can be modeled by $y = -0.00211x^2 + 1.06x$, where x is the distance (in meters) from the left pylons and y is the height (in meters) of the arch above the water. For what distances x is the arch above the road?



46. **PROBLEM SOLVING** The number T of teams that have participated in a robot-building competition for high-school students over a recent period of time x (in years) can be modeled by

$$T(x) = 17.155x^2 + 193.68x + 235.81, 0 \leq x \leq 6.$$

After how many years is the number of teams greater than 1000? Justify your answer.



47. **PROBLEM SOLVING** A study found that a driver's reaction time $A(x)$ to audio stimuli and his or her reaction time $V(x)$ to visual stimuli (both in milliseconds) can be modeled by

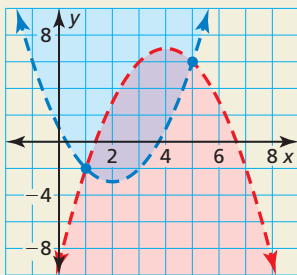
$$A(x) = 0.0051x^2 - 0.319x + 15, 16 \leq x \leq 70$$

$$V(x) = 0.005x^2 - 0.23x + 22, 16 \leq x \leq 70$$

where x is the age (in years) of the driver.

- a. Write an inequality that you can use to find the x -values for which $A(x)$ is less than $V(x)$.
 b. Use a graphing calculator to solve the inequality $A(x) < V(x)$. Describe how you used the domain $16 \leq x \leq 70$ to determine a reasonable solution.
 c. Based on your results from parts (a) and (b), do you think a driver would react more quickly to a traffic light changing from green to yellow or to the siren of an approaching ambulance? Explain.

48. **HOW DO YOU SEE IT?** The graph shows a system of quadratic inequalities.



- a. Identify two solutions of the system.
- b. Are the points $(1, -2)$ and $(5, 6)$ solutions of the system? Explain.
- c. Is it possible to change the inequality symbol(s) so that one, but not both of the points in part (b), is a solution of the system? Explain.
49. **MODELING WITH MATHEMATICS** The length L (in millimeters) of the larvae of the black porgy fish can be modeled by

$$L(x) = 0.00170x^2 + 0.145x + 2.35, 0 \leq x \leq 40$$

where x is the age (in days) of the larvae. Write and solve an inequality to find at what ages a larva's length tends to be greater than 10 millimeters. Explain how the given domain affects the solution.

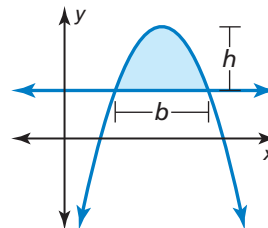


50. **MAKING AN ARGUMENT** You claim the system of inequalities below, where a and b are real numbers, has no solution. Your friend claims the system will always have at least one solution. Who is correct? Explain.

$$y < (x + a)^2$$

$$y < (x + b)^2$$

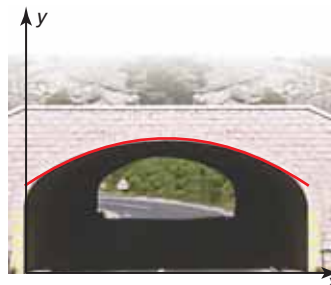
51. **MATHEMATICAL CONNECTIONS** The area A of the region bounded by a parabola and a horizontal line can be modeled by $A = \frac{2}{3}bh$, where b and h are as defined in the diagram. Find the area of the region determined by each pair of inequalities.



- a. $y \leq -x^2 + 4x$
 $y \geq 0$
- b. $y \geq x^2 - 4x - 5$
 $y \leq 7$

52. **THOUGHT PROVOKING** Draw a company logo that is created by the intersection of two quadratic inequalities. Justify your answer.

53. **REASONING** A truck that is 11 feet tall and 7 feet wide is traveling under an arch. The arch can be modeled by $y = -0.0625x^2 + 1.25x + 5.75$, where x and y are measured in feet.



- a. Will the truck fit under the arch? Explain.
- b. What is the maximum width that a truck 11 feet tall can have and still make it under the arch?
- c. What is the maximum height that a truck 7 feet wide can have and still make it under the arch?

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Graph the function. Label the x -intercept(s) and the y -intercept. (Section 2.2)

54. $f(x) = (x + 7)(x - 9)$

55. $g(x) = (x - 2)^2 - 4$

56. $h(x) = -x^2 + 5x - 6$

Find the minimum value or maximum value of the function. Then describe where the function is increasing and decreasing. (Section 2.2)

57. $f(x) = -x^2 - 6x - 10$

58. $h(x) = \frac{1}{2}(x + 2)^2 - 1$

59. $f(x) = -(x - 3)(x + 7)$

60. $h(x) = x^2 + 3x - 18$

3.4–3.6 What Did You Learn?

Core Vocabulary

Quadratic Formula, *p.* 122

discriminant, *p.* 124

system of nonlinear equations, *p.* 132

quadratic inequality in two variables, *p.* 140

quadratic inequality in one variable, *p.* 142

Core Concepts

Section 3.4

Solving Equations Using the Quadratic Formula, *p.* 122

Analyzing the Discriminant of $ax^2 + bx + c = 0$, *p.* 124

Methods for Solving Quadratic Equations, *p.* 125

Modeling Launched Objects, *p.* 126

Section 3.5

Solving Systems of Nonlinear Equations, *p.* 132

Solving Equations by Graphing, *p.* 135

Section 3.6

Graphing a Quadratic Inequality in Two Variables, *p.* 140

Solving Quadratic Inequalities in One Variable, *p.* 142

Mathematical Practices

1. How can you use technology to determine whose rocket lands first in part (b) of Exercise 65 on page 129?
2. What question can you ask to help the person avoid making the error in Exercise 54 on page 138?
3. Explain your plan to find the possible widths of the fountain in Exercise 44 on page 145.

Performance Task

Algebra in Genetics: The Hardy-Weinberg Law

Some people have attached earlobes, the recessive trait. Some people have free earlobes, the dominant trait. What percent of people carry both traits?

To explore the answers to this question and more, go to BigIdeasMath.com.



3.1 Solving Quadratic Equations (pp. 93–102)

In a physics class, students must build a Rube Goldberg machine that drops a ball from a 3-foot table. Write a function h (in feet) of the ball after t seconds. How long is the ball in the air?

The initial height is 3, so the model is $h = -16t^2 + 3$. Find the zeros of the function.

$$h = -16t^2 + 3 \quad \text{Write the function.}$$

$$0 = -16t^2 + 3 \quad \text{Substitute 0 for } h.$$

$$-3 = -16t^2 \quad \text{Subtract 3 from each side.}$$

$$\frac{-3}{-16} = t^2 \quad \text{Divide each side by } -16.$$

$$\pm\sqrt{\frac{3}{16}} = t \quad \text{Take square root of each side.}$$

$$\pm 0.3 \approx t \quad \text{Use a calculator.}$$

► Reject the negative solution, -0.3 , because time must be positive. The ball will fall for about 0.3 second before it hits the ground.

1. Solve $x^2 - 2x - 8 = 0$ by graphing.

Solve the equation using square roots or by factoring.

2. $3x^2 - 4 = 8$

3. $x^2 + 6x - 16 = 0$

4. $2x^2 - 17x = -30$

5. A rectangular enclosure at the zoo is 35 feet long by 18 feet wide. The zoo wants to double the area of the enclosure by adding the same distance x to the length and width. Write and solve an equation to find the value of x . What are the dimensions of the enclosure?

3.2 Complex Numbers (pp. 103–110)

Perform each operation. Write the answer in standard form.

a. $(3 - 6i) - (7 + 2i) = (3 - 7) + (-6 - 2)i$
 $= -4 - 8i$

b. $5i(4 + 5i) = 20i + 25i^2$
 $= 20i + 25(-1)$
 $= -25 + 20i$

6. Find the values x and y that satisfy the equation $36 - yi = 4x + 3i$.

Perform the operation. Write the answer in standard form.

7. $(-2 + 3i) + (7 - 6i)$

8. $(9 + 3i) - (-2 - 7i)$

9. $(5 + 6i)(-4 + 7i)$

10. Solve $7x^2 + 21 = 0$.

11. Find the zeros of $f(x) = 2x^2 + 32$.

3.3 Completing the Square (pp. 111–118)

Solve $x^2 + 12x + 8 = 0$ by completing the square.

$$x^2 + 12x + 8 = 0$$

Write the equation.

$$x^2 + 12x = -8$$

Write left side in the form $x^2 + bx$.

$$x^2 + 12x + 36 = -8 + 36$$

Add $\left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 36$ to each side.

$$(x + 6)^2 = 28$$

Write left side as a binomial squared.

$$x + 6 = \pm \sqrt{28}$$

Take square root of each side.

$$x = -6 \pm \sqrt{28}$$

Subtract 6 from each side.

$$x = -6 \pm 2\sqrt{7}$$

Simplify radical.

► The solutions are $x = -6 + 2\sqrt{7}$ and $x = -6 - 2\sqrt{7}$.

12. An employee at a local stadium is launching T-shirts from a T-shirt cannon into the crowd during an intermission of a football game. The height h (in feet) of the T-shirt after t seconds can be modeled by $h = -16t^2 + 96t + 4$. Find the maximum height of the T-shirt.

Solve the equation by completing the square.

13. $x^2 + 16x + 17 = 0$

14. $4x^2 + 16x + 25 = 0$

15. $9x(x - 6) = 81$

16. Write $y = x^2 - 2x + 20$ in vertex form. Then identify the vertex.

3.4 Using the Quadratic Formula (pp. 121–130)

Solve $-x^2 + 4x = 5$ using the Quadratic Formula.

$$-x^2 + 4x = 5$$

Write the equation.

$$-x^2 + 4x - 5 = 0$$

Write in standard form.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)}$$

$a = -1, b = 4, c = -5$

$$x = \frac{-4 \pm \sqrt{-4}}{-2}$$

Simplify.

$$x = \frac{-4 \pm 2i}{-2}$$

Write in terms of i .

$$x = 2 \pm i$$

Simplify.

► The solutions are $2 + i$ and $2 - i$.

Solve the equation using the Quadratic Formula.

17. $-x^2 + 5x = 2$

18. $2x^2 + 5x = 3$

19. $3x^2 - 12x + 13 = 0$

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

20. $-x^2 - 6x - 9 = 0$

21. $x^2 - 2x - 9 = 0$

22. $x^2 + 6x + 5 = 0$

3.5 Solving Nonlinear Systems (pp. 131–138)

Solve the system by elimination.

$$2x^2 - 8x + y = -5$$

Equation 1

$$2x^2 - 16x - y = -31$$

Equation 2

Add the equations to eliminate the y -term and obtain a quadratic equation in x .

$$2x^2 - 8x + y = -5$$

$$2x^2 - 16x - y = -31$$

$$4x^2 - 24x = -36$$

Add the equations.

$$4x^2 - 24x + 36 = 0$$

Write in standard form.

$$x^2 - 6x + 9 = 0$$

Divide each side by 4.

$$(x - 3)^2 = 0$$

Perfect Square Trinomial Pattern

$$x = 3$$

Zero-Product Property

To solve for y , substitute $x = 3$ in Equation 1 to obtain $y = 1$.

► So, the solution is $(3, 1)$.

Solve the system by any method. Explain your choice of method.

23. $2x^2 - 2 = y$

24. $x^2 - 6x + 13 = y$

25. $x^2 + y^2 = 4$

$-2x + 2 = y$

$-y = -2x + 3$

$-15x + 5 = 5y$

26. Solve $-3x^2 + 5x - 1 = 5x^2 - 8x - 3$ by graphing.

3.6 Quadratic Inequalities (pp. 139–146)

Graph the system of quadratic inequalities.

$$y > x^2 - 2$$

Inequality 1

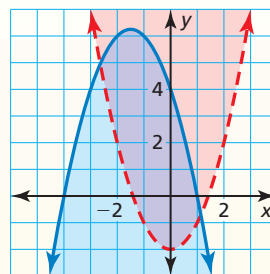
$$y \leq -x^2 - 3x + 4$$

Inequality 2

Step 1 Graph $y > x^2 - 2$. The graph is the red region inside (but not including) the parabola $y = x^2 - 2$.

Step 2 Graph $y \leq -x^2 - 3x + 4$. The graph is the blue region inside and including the parabola $y = -x^2 - 3x + 4$.

Step 3 Identify the purple region where the two graphs overlap. This region is the graph of the system.



Graph the inequality.

27. $y > x^2 + 8x + 16$

28. $y \geq x^2 + 6x + 8$

29. $x^2 + y \leq 7x - 12$

Graph the system of quadratic inequalities.

30. $x^2 - 4x + 8 > y$

31. $2x^2 - x \geq y - 5$

32. $-3x^2 - 2x \leq y + 1$

$-x^2 + 4x + 2 \leq y$

$0.5x^2 > y - 2x - 1$

$-2x^2 + x - 5 > -y$

Solve the inequality.

33. $3x^2 + 3x - 60 \geq 0$

34. $-x^2 - 10x < 21$

35. $3x^2 + 2 \leq 5x$

3 Chapter Test

Solve the equation using any method. Provide a reason for your choice.

1. $0 = x^2 + 2x + 3$

2. $6x = x^2 + 7$

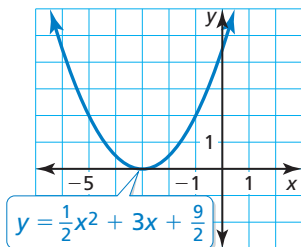
3. $x^2 + 49 = 85$

4. $(x + 4)(x - 1) = -x^2 + 3x + 4$

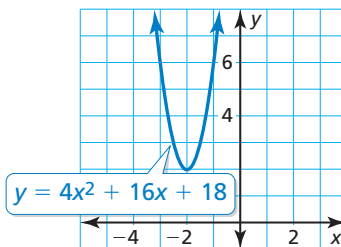
Explain how to use the graph to find the number and type of solutions of the quadratic equation.

Justify your answer by using the discriminant.

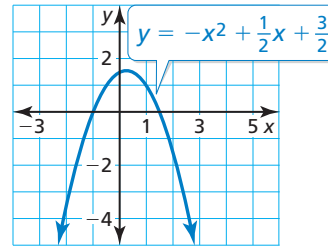
5. $\frac{1}{2}x^2 + 3x + \frac{9}{2} = 0$



6. $4x^2 + 16x + 18 = 0$



7. $-x^2 + \frac{1}{2}x + \frac{3}{2} = 0$



Solve the system of equations or inequalities.

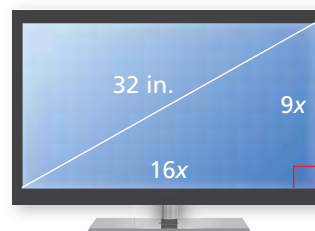
8. $x^2 + 66 = 16x - y$
 $2x - y = 18$

9. $y \geq \frac{1}{4}x^2 - 2$
 $y < -(x + 3)^2 + 4$

10. $0 = x^2 + y^2 - 40$
 $y = x + 4$

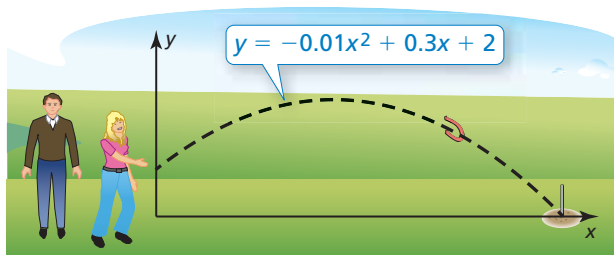
11. Write $(3 + 4i)(4 - 6i)$ as a complex number in standard form.

12. The *aspect ratio* of a widescreen TV is the ratio of the screen's width to its height, or 16 : 9. What are the width and the height of a 32-inch widescreen TV? Justify your answer. (*Hint:* Use the Pythagorean Theorem and the fact that TV sizes refer to the diagonal length of the screen.)



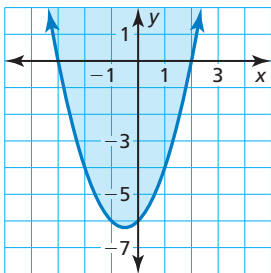
13. The shape of the Gateway Arch in St. Louis, Missouri, can be modeled by $y = -0.0063x^2 + 4x$, where x is the distance (in feet) from the left foot of the arch and y is the height (in feet) of the arch above the ground. For what distances x is the arch more than 200 feet above the ground? Justify your answer.

14. You are playing a game of horseshoes. One of your tosses is modeled in the diagram, where x is the horseshoe's horizontal position (in feet) and y is the corresponding height (in feet). Find the maximum height of the horseshoe. Then find the distance the horseshoe travels. Justify your answer.



3 Cumulative Assessment

1. The graph of which inequality is shown?



- (A) $y > x^2 + x - 6$
 (B) $y \geq x^2 + x - 6$
 (C) $y > x^2 - x - 6$
 (D) $y \geq x^2 - x - 6$

2. Classify each function by its function family. Then describe the transformation of the parent function.

a. $g(x) = x + 4$

b. $h(x) = 5$

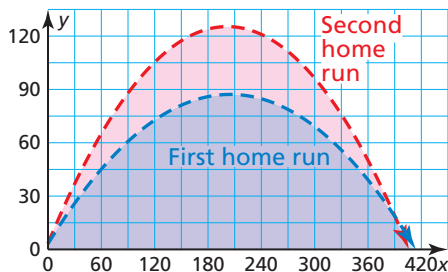
c. $h(x) = x^2 - 7$

d. $g(x) = -|x + 3| - 9$

e. $g(x) = \frac{1}{4}(x - 2)^2 - 1$

f. $h(x) = 6x + 11$

3. Two baseball players hit back-to-back home runs. The path of each home run is modeled by the parabolas below, where x is the horizontal distance (in feet) from home plate and y is the height (in feet) above the ground. Choose the correct symbol for each inequality to model the possible locations of the top of the outfield wall.



First home run: y $-0.002x^2 + 0.82x + 3.1$

Second home run: y $0.003x^2 + 1.21x + 3.3$

4. You claim it is possible to make a function from the given values that has an axis of symmetry at $x = 2$. Your friend claims it is possible to make a function that has an axis of symmetry at $x = -2$. What values can you use to support your claim? What values support your friend's claim?

Your claim
$f(x) =$ <input type="text"/> $x^2 +$ <input type="text"/> $x + 8$

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
8	-6	-2	0

Your friend's claim
$f(x) =$ <input type="text"/> $x^2 +$ <input type="text"/> $x + 8$

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
1	3	5	12

5. Which of the following values are x -coordinates of the solutions of the system?

$$y = x^2 - 6x + 14$$

$$y = 2x + 7$$

-9	-7	-5	-3	-1
1	3	5	7	9

6. The table shows the altitudes of a hang glider that descends at a constant rate. How long will it take for the hang glider to descend to an altitude of 100 feet? Justify your answer.

Time (seconds), t	Altitude (feet), y
0	450
10	350
20	250
30	150

(A) 25 seconds

(B) 35 seconds

(C) 45 seconds

(D) 55 seconds

7. Use the numbers and symbols to write the expression $x^2 + 16$ as the product of two binomials. Some may be used more than once. Justify your answer.

+	-	16	x)
4	i	2	8	(

8. Choose values for the constants h and k in the equation $x = \frac{1}{4}(y - k)^2 + h$ so that each statement is true.

a. The graph of $x = \frac{1}{4}(y - \square)^2 + \square$ is a parabola with its vertex in the second quadrant.

b. The graph of $x = \frac{1}{4}(y - \square)^2 + \square$ is a parabola with its focus in the first quadrant.

c. The graph of $x = \frac{1}{4}(y - \square)^2 + \square$ is a parabola with its focus in the third quadrant.

9. Which of the following graphs represent a perfect square trinomial? Write each function in vertex form by completing the square.

